The Simple ACT® Enhanced Math Algebra Cheat Sheet

Expressions & Exponent Rules

Simplifying Expressions

Like terms: same variable & same power combine ($3x^2 + 5x^2 = 8x^2$); $3x^2 + 5x$ can't combine.

Distribute to every term: $-2(x - 4) = -2x + 8$. A sign error on the 2nd term is the #1 ACT trap.

Exponent Rules

Rule	Example
$x^a \cdot x^b = x^{a+b}$	$x^3 \cdot x^4 = x^7$
$x^a / x^b = x^{a-b}$	$x^5 / x^2 = x^3$
$(x^a)^b = x^{ab}$	$(x^2)^3 = x^6$
$(xy)^a = x^a y^a$	$(2x)^3 = 8x^3$
$x^0 = 1\ (x \neq 0)$	$7^0 = 1$
$x^{-a} = 1/x^a$	$2^{-3} = 1/8$

Power of a sum: $(x + y)^2 \neq x^2 + y^2$. Expand it: $(x + y)^2 = x^2 + 2xy + y^2$ (don't drop the middle 2xy).

Same base: rewrite $4^x = 2^{2x}$ & $27 = 3^3$, then set the exponents equal & solve for x.

Fast Tricks

Plug in numbers: when answers are expressions, pick x = 2 & test each choice; avoid 0 & 1.

Negative base: $(-3)^2 = 9$ but $-3^2 = -9$ b/c the exponent goes before the minus.

⚠ Exponents only combine when you multiply/divide like bases. $x^2 + x^3$ stays as is.

✓ Calculators are allowed on every question. Use yours to check exponent math.

Linear Equations & Inequalities

Solving Linear Equations

Goal: isolate x. Undo + / − first, then × / ÷. Do the same thing to both sides at every step, then check.

Variables on both sides: collect x terms on 1 side, constants on the other ($5x - 3 = 2x + 9 \rightarrow x = 4$).

Fractions: multiply both sides by the LCD first: $x/3 + x/4 = 7 \rightarrow 4x + 3x = 84 \rightarrow x = 12$.

Result	Meaning	Example
x = number	1 solution	2x = 6
0 = 0	Infinitely many	2x + 2 = 2(x + 1)
0 = 5	No solution	x + 1 = x + 6

Literal Equations

Solve for a letter: treat other letters as numbers. $A = \frac{1}{2}bh \rightarrow h = 2A/b$; $P = 2l + 2w \rightarrow w = (P - 2l)/2$.

Inequalities

Flip the sign when you multiply or divide by a negative: $-2x > 6 \rightarrow x < -3$. Adding never flips.

Compound inequality: $-3 < 2x + 1 \leq 9 \rightarrow$ do each step to all 3 parts: $-4 < 2x \leq 8 \rightarrow -2 < x \leq 4$.

Graph: open circle for < or >, closed for ≤ or ≥.

Absolute Value

$|x - 3| = 5$ → $x - 3 = 5$ or $x - 3 = -5 \rightarrow x = 8$ or $x = -2$. $|x| = -4$ has no solution (never negative).

$|x| < a$ → $-a < x < a$ (between). **$|x| > a$** → $x < -a$ or $x > a$ (outside). Same rules apply to $|x - h|$.

⚠ Forgetting to flip the sign when dividing by a negative is the top inequality mistake.

✓ Stuck? Backsolve: plug each answer choice into the equation, starting in the middle.

Lines, Slope & Linear Models

Slope

Slope $m = (y_2 - y_1)/(x_2 - x_1)$ = rise/run.

Line	Slope
Rises left → right	Positive
Falls left → right	Negative
Horizontal (y = c)	0
Vertical (x = c)	Undefined

Forms of a Line

Form	Equation	Best for
Slope-int.	$y = mx + b$	Graphing
Point-slope	$y - y_1 = m(x - x_1)$	1 pt + slope
Standard	$Ax + By = C$	Intercepts

Standard form slope: $m = -A/B$. x-int: set y = 0; y-int: set x = 0. $2x + 3y = 12 \rightarrow m = -2/3$.

Parallel: same slope. **Perpendicular**: negative reciprocal slopes (2 & −1/2); product = −1.

Linear Models

In context: $y = mx + b \rightarrow$ m = rate per unit (cost per item), b = starting value (flat fee).

Example: taxi $C = 2.50 + 1.75m \rightarrow$ $2.50 base fee, $1.75 per mile; 10 miles costs $20.

Table of values: if y changes by the same amount each step, the relationship is linear.

⚠ Slope is rise over RUN. Flipping it to $\Delta x/\Delta y$ gives the reciprocal, a common trap.

✓ Interpret slope w/ units & context: "$1.75 per mile," not just the number "1.75."

Systems of Equations

Solving Methods

Substitution: solve 1 equation for a variable, plug into the other. Best when a coefficient is 1.

Elimination: scale equations so 1 variable cancels when you add/subtract. Best w/ standard form.

Example: $2x + y = 7$ & $x - y = 2 \rightarrow$ add: $3x = 9 \rightarrow x = 3$, then $y = 1$. Check (3, 1) in both equations.

Graphing: the solution is the point where the lines cross; use a calculator for decimals.

Number of Solutions

Lines	Slopes	Solutions
Intersect	Different	Exactly 1
Parallel	Same, diff. b	None
Same line	Same, same b	Infinite

"No solution": set slopes equal & intercepts unequal, then solve for the constant.

Word Problems & Inequalities

Set up: 1 equation per fact. Tickets: $a + s = 200$ (count) & $8a + 5s = 1{,}300$ (money) $\rightarrow a = 100$.

Mixture problems: (amount)(rate) + (amount)(rate) = total; keep units the same (cents or dollars).

Nonlinear: line + parabola → substitute & solve the quadratic; 0, 1 or 2 solutions (crossing points).

Systems of inequalities: solution = overlap of shaded regions; dashed line for < or >.

⚠ Solve for what's asked. If the question wants x + y, you may not need x & y separately.

✓ Asked for x + y? Try adding the 2 equations first; it may give x + y right away.

Polynomials & Factoring

Operations

Add/subtract: combine like terms; distribute the minus to every term of the 2nd polynomial.

Multiply: every term × every term (FOIL): $(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$.

Factoring Patterns

Pattern	Factored form
GCF	$6x^2 + 9x = 3x(2x + 3)$
Diff. of squares	$a^2 - b^2 = (a + b)(a - b)$
Perfect square	$a^2 + 2ab + b^2 = (a + b)^2$
Trinomial	$x^2 + 5x + 6 = (x + 2)(x + 3)$
Grouping	Pair terms, pull GCF

$x^2 + bx + c$: find 2 numbers that multiply to c & add to b. Always pull out a GCF first, if there is one.

Degree: the highest power = the most real zeros a polynomial can have ($x^3 \rightarrow$ up to 3).

Zeros & Factors

Zero product property: if $(x - 2)(x + 4) = 0$, then $x = 2$ or $x = -4$. Only works when 1 side = 0.

Factor ↔ zero: $(x - k)$ is a factor ⇔ $p(k) = 0$ ⇔ $x = k$ is an x-intercept of the graph of $y = p(x)$.

Remainder theorem: dividing p(x) by $(x - k)$ leaves remainder p(k); $p(k) = 0 \rightarrow (x - k)$ is a factor.

⚠ Zero x = 3 means factor (x − 3), not (x + 3). The sign always flips between a zero & its factor.

✓ Check factoring by multiplying the factors back out. It catches sign errors fast.

Quadratic Equations

3 Ways to Solve

1. Factor: set = 0, factor, use zero product. Fastest when it factors nicely over the integers.

2. Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ for $ax^2 + bx + c = 0$. Always works; memorize it.

3. Square root: $(x - 3)^2 = 16 \rightarrow x - 3 = \pm 4 \rightarrow x = 7$ or -1. Don't forget the ±; there are 2 answers.

Example: $2x^2 - 3x - 2 = 0 \rightarrow (2x + 1)(x - 2) = 0 \rightarrow x = -1/2$ or $x = 2$ (check by plugging in).

Sum & product of roots: sum = $-b/a$, product = c/a. No need to solve the quadratic first.

Max/min: the vertex gives the highest or lowest point (max height of a ball, lowest cost).

Discriminant $b^2 - 4ac$

Value	Real solutions	Graph
> 0	2	Crosses x-axis 2×
= 0	1 (double)	Touches once
< 0	0 (2 complex)	Misses x-axis

Forms of a Parabola

Form	Shows
$y = ax^2 + bx + c$	y-int c; vertex $x = -b/2a$
$y = a(x - h)^2 + k$	Vertex (h, k)
$y = a(x - p)(x - q)$	Zeros p & q

a > 0 opens up (min); **a < 0** opens down (max). Larger $|a| \rightarrow$ narrower parabola.

⚠ In the formula, −b means the opposite of b. If b = −6, then −b = +6. Watch double negatives.

Radicals & Rational Expressions

Radicals

Simplify: pull out perfect squares: $\sqrt{72} = \sqrt{(36 \cdot 2)} = 6\sqrt{2}$. Rule: $\sqrt{(ab)} = \sqrt{a} \cdot \sqrt{b}$ (never split sums).

Rationalize: $6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$.

Add only like radicals: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$. **Multiply** freely: $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$, $\sqrt{5} \cdot \sqrt{5} = 5$.

Fractional exponents: $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$; $16^{3/4} = (\sqrt[4]{16})^3 = 8$. Top = power, bottom = root.

Radical Equations

Steps: isolate the radical, square both sides, solve, then CHECK every answer in the original.

Example: $\sqrt{(x + 7)} = x - 5 \rightarrow x + 7 = x^2 - 10x + 25 \rightarrow x = 9$ or 2; only x = 9 works ($x = 2$ gives $3 = -3$).

Extraneous solution: value from squaring that fails the original (makes a radical equal a negative).

Cancel factors, not terms: $(x + 2)(x - 1)/(x + 2) = x - 1$, but $(x + 2)/(x + 5)$ won't reduce.

Rational Expressions & Equations

Task	Move
Simplify	Factor, cancel factors
Multiply	Factor, cancel, multiply
Divide	Flip 2nd, multiply
Add/subtract	Common denominator

Solve: multiply both sides by the LCD; check answers that make any denominator 0 (excluded).

⚠ Always check radical & rational solutions. The ACT lists extraneous answers as choices.

Exponential Equations & Modeling

Exponential Models

Growth/decay: $y = a(1 \pm r)^t$. a = start, r = % rate as a decimal, t = time periods (years, months, etc.).

Situation	Model
Grows 5%/yr	$a(1.05)^t$
Shrinks 20%/yr	$a(0.80)^t$
Doubles every 3 yr	$a \cdot 2^{t/3}$
Halves every 10 yr	$a(\frac{1}{2})^{t/10}$

Base b: $b > 1 \rightarrow$ growth; $0 < b < 1 \rightarrow$ decay (nears 0, never reaches it). y-int = a, the value at t = 0.

Solving Exponential Equations

Same base: $3^{x+1} = 81 = 3^4 \rightarrow x + 1 = 4 \rightarrow x = 3$. Rewrite both sides as powers of 1 base first.

Different bases: graph both sides on the calculator & find the intersection.

Linear vs. Exponential

Linear: adds the same amount each step.

Exponential: multiplies by the same factor.

Translating words: "is" → =, "of" → ×, "per" → ÷, "more than" → +, "less than" reverses order.

"5 less than x" = $x - 5$, not $5 - x$. "x decreased by 30%" = 0.7x, not $x - 0.3$ or $x - 30$.

⚠ A 20% decrease uses 0.80 as the base, not 0.20. Base = 1 − rate for decay.

✓ Table test: same difference → linear; same ratio → exponential. Check 3 rows.