# The Ultimate ACT® Enhanced Math Algebra Cheat Sheet



## Expressions & Exponent Rules

### Simplifying Expressions

**Like terms**: same variable & same power combine ($3x^2 + 5x^2 = 8x^2$); $3x^2 + 5x$ can't combine at all: different powers.

**Distribute** to every term: $-2(x - 4) = -2x + 8$. A sign error on the 2nd term is the #1 ACT trap on these Qs.

**Order of operations**: PEMDAS; multiply/divide & add/subtract each go left → right, not M before D or A before S.

### Exponent Rules

| Rule | Example |
|---|---|
| $x^a \cdot x^b = x^{a+b}$ | $x^3 \cdot x^4 = x^7$ |
| $x^a / x^b = x^{a-b}$ | $x^5 / x^2 = x^3$ |
| $(x^a)^b = x^{ab}$ | $(x^2)^3 = x^6$ |
| $(xy)^a = x^a y^a$ | $(2x)^3 = 8x^3$ |
| $x^0 = 1\ (x \neq 0)$ | $7^0 = 1$ |
| $x^{-a} = 1/x^a$ | $2^{-3} = 1/8$ |
| $x^{a/b} = (\sqrt[b]{x})^a$ | $8^{2/3} = 4$ |

**Power of a sum**: $(x + y)^2 \neq x^2 + y^2$. Expand it: $(x + y)^2 = x^2 + 2xy + y^2$ (don't drop the middle $2xy$ term).

**Negative base**: $(-3)^2 = 9$ but $-3^2 = -9$ (exponent goes first); type $(-3)^2$ w/ parentheses on a calculator.

### Fast Tricks

**Same base**: rewrite $4^x = 2^{2x}$, $27 = 3^3$, $1/9 = 3^{-2}$, then set the exponents equal to each other & solve.

**Plug in numbers**: when answers are expressions, pick $x = 2$ & test each choice; avoid 0 & 1 (too many ties).

⚠ Exponents only combine when you multiply/divide like bases. $x^2 + x^3$ stays exactly as it is; it's not $x^5$.

✓ Calculator is allowed on every question. Use it to check exponent arithmetic, not to replace algebra.

## Linear Equations & Inequalities

### Solving Linear Equations

**Goal**: isolate x. Undo + / − first, then × / ÷. Do the same thing to both sides at every step, then check.

**Fractions**: multiply both sides by the LCD to clear them first ($x/3 + x/4 = 7 \rightarrow 4x + 3x = 84 \rightarrow x = 12$).

**Variables on both sides**: collect x terms on 1 side, constants on the other ($5x - 3 = 2x + 9 \rightarrow x = 4$).

| Result | Meaning | Example |
|---|---|---|
| x = number | 1 solution | $2x = 6$ |
| 0 = 0 | Infinitely many | $2x + 2 = 2(x + 1)$ |
| 0 = 5 | No solution | $x + 1 = x + 6$ |

### Literal Equations

**Solve for a letter**: treat other letters as numbers. $A = \frac{1}{2}bh \rightarrow h = 2A/b$; $P = 2l + 2w \rightarrow w = (P - 2l)/2 = P/2 - l$.

### Inequalities

**Flip the sign** when you multiply or divide by a negative: $-2x > 6 \rightarrow x < -3$. Adding/subtracting never flips.

**Compound inequality**: $-3 < 2x + 1 \leq 9 \rightarrow$ do each step to all 3 parts: $-4 < 2x \leq 8 \rightarrow -2 < x \leq 4$. Keep the order.

**Infinite or none**: $ax + b > ax + c$ is always true if $b > c$, never true if $b \leq c$ (x cancels out entirely; no x left).

**Graph**: open circle for < or >, closed for ≤ or ≥; "and" = between, "or" = 2 rays pointing outward.

### Absolute Value

$|x - 3| = 5 \rightarrow$ split: $x - 3 = 5$ or $x - 3 = -5 \rightarrow x = 8$ or $x = -2$. $|x| = -4$ has no solution ($|x|$ is never negative).

$|x| < a \rightarrow -a < x < a$ (between). $|x| > a \rightarrow x < -a$ or $x > a$ (outside). Same rules apply to $|x - h|$.

⚠ Forgetting to flip the inequality when dividing by a negative is the most common inequality miss.

✓ Stuck? Backsolve: plug each answer choice into the original equation, starting w/ the middle value.

## Lines, Slope & Linear Models

### Slope

**Slope** $m = (y_2 - y_1)/(x_2 - x_1)$ = rise/run = rate of change.

| Line | Slope |
|---|---|
| Rises left → right | Positive |
| Falls left → right | Negative |
| Horizontal (y = c) | 0 |
| Vertical (x = c) | Undefined |

### Forms of a Line

| Form | Equation | Best for |
|---|---|---|
| Slope-int. | $y = mx + b$ | Graphing |
| Point-slope | $y - y_1 = m(x - x_1)$ | 1 pt + slope |
| Standard | $Ax + By = C$ | Intercepts |

**Standard form slope**: $m = -A/B$. x-int: set $y = 0$; y-int: set $x = 0$. $2x + 3y = 12 \rightarrow m = -2/3$, intercepts (6, 0) & (0, 4).

**Parallel**: same slope. **Perpendicular**: negative reciprocal slopes (2 & −1/2); product = −1. Vertical ⊥ horizontal.

### Linear Models

**In context**: $y = mx + b \rightarrow m$ = rate per unit (cost per item), b = starting value (the flat fee when $x = 0$).

**Example**: taxi C = 2.50 + 1.75m → $2.50 base fee, $1.75 per mile; 10 miles costs $2.50 + $17.50 = $20.

**Table of values**: constant change in y for equal steps in x → linear; slope = Δy/Δx from any 2 rows.

**Direct variation**: $y = kx$ is a line through the origin; $k = y/x$ stays constant (double x → double y).

**Rate questions**: "how much does y change when x increases by 3?" → 3 × slope ($m = 4 \rightarrow y$ ↑ by 12).

⚠ Slope is rise over RUN. Swapping to Δx/Δy gives the reciprocal, & it's always an answer choice.

✓ "Interpret the slope" answers always use units & context: "$1.75 per mile," not just "1.75" or "the rate."

## Systems of Equations

### Solving Methods

**Substitution**: solve 1 equation for a variable, plug into the other. Best when a coefficient is 1 or −1.

**Elimination**: scale equations so 1 variable cancels when you add/subtract. Best w/ standard form ($Ax + By = C$).

**Example**: $2x + y = 7$ & $x - y = 2 \rightarrow$ add: $3x = 9 \rightarrow x = 3$, then $y = 1$. Check (3, 1) in both equations before picking.

**Scaling**: $3x + 2y = 8$ & $2x - y = 3 \rightarrow$ multiply the 2nd by 2 & add: $7x = 14 \rightarrow x = 2$, $y = 1$ (check: 6 + 2 = 8).

**Graphing**: solution = intersection point; use the calculator's graph when choices are decimals or ugly fractions.

### Number of Solutions

| Lines | Slopes | Solutions |
|---|---|---|
| Intersect | Different | Exactly 1 |
| Parallel | Same, diff. b | None |
| Same line | Same, same b | Infinite |

**"No solution" questions**: set slopes equal & intercepts unequal, then solve for the unknown constant.

### Word Problems & Inequalities

**Set up**: 1 equation per fact. Tickets: $a + s = 200$ (count) & $8a + 5s = 1{,}300$ (money) → $a = 100$, $s = 100$.

**Mixture/value**: (amount)(rate) + (amount)(rate) = total; keep units the same in each equation (cents vs. dollars).

**Systems of inequalities**: solution = overlap of shaded regions; dashed line for < or >, solid for ≤ or ≥.

**Nonlinear systems**: line + parabola → substitute & solve the quadratic; can have 0, 1 or 2 solutions.

⚠ Solve for what's asked. If the question wants x + y, you may not need x & y separately; check the last line.

✓ Asked for x + y? Try adding the 2 equations first; it often gives a multiple of x + y right away ($3x + 3y = 12 \rightarrow 4$).

## Polynomials & Factoring

### Operations

**Add/subtract**: combine like terms; distribute the minus to every term of the 2nd polynomial, not just the first.

**Multiply**: every term × every term (FOIL for 2 binomials): $(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$.

### Factoring Patterns

| Pattern | Factored form |
|---|---|
| GCF | $6x^2 + 9x = 3x(2x + 3)$ |
| Diff. of squares | $a^2 - b^2 = (a + b)(a - b)$ |
| Perfect square | $a^2 + 2ab + b^2 = (a + b)^2$ |
| Trinomial | $x^2 + 5x + 6 = (x + 2)(x + 3)$ |
| Grouping | Pair terms, pull GCF |

**$x^2 + bx + c$**: find 2 numbers that multiply to c & add to b. Always pull out a GCF first, if there is one.

**$ax^2 + bx + c$** ($a \neq 1$): AC method: find numbers multiplying to ac & adding to b, then split the middle & group in pairs.

### Zeros & Factors

**Zero product property**: if $(x - 2)(x + 4) = 0$, then $x = 2$ or $x = -4$. Only works when 1 side = 0, never = 5.

**Factor ↔ zero**: $(x - k)$ is a factor ⇔ $p(k) = 0$ ⇔ $x = k$ is an x-intercept of the graph of $y = p(x)$. All 3 are the same fact.

**Remainder theorem**: $p(x) \div (x - k)$ leaves remainder $p(k)$. $p(2) = 0 \rightarrow (x - 2)$ divides evenly, no long division.

**Degree**: highest power = max number of real zeros; end behavior is set by the leading term (sign & degree).

**Sum/difference of cubes**: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$; rare but appears on hard questions ($x^3 - 8$).

⚠ Zero $x = 3$ means factor $(x - 3)$, not $(x + 3)$. The sign always flips between a zero & its factor.

✓ Factoring check: multiply your factors back out. It takes 10 seconds & catches sign errors before they cost you.

## Quadratic Equations

### 3 Ways to Solve

**1. Factor**: set = 0, factor, use zero product. Fastest when it factors nicely over the integers; try it 1st.

**2. Quadratic formula**: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ for $ax^2 + bx + c = 0$. Always works; memorize it (no formula sheet).

**3. Square root**: $(x - 3)^2 = 16 \rightarrow x - 3 = \pm 4 \rightarrow x = 7$ or $-1$. Don't forget the ±; there are 2 answers.

**Example**: $2x^2 - 3x - 2 = 0 \rightarrow (2x + 1)(x - 2) = 0 \rightarrow x = -1/2$ or $x = 2$; discriminant $= 9 + 16 = 25$ (2 roots).

### Discriminant $b^2 - 4ac$

| Value | Real solutions | Graph |
|---|---|---|
| > 0 | 2 | Crosses x-axis 2× |
| = 0 | 1 (double) | Touches once |
| < 0 | 0 (2 complex) | Misses x-axis |

### Forms of a Parabola

| Form | Shows |
|---|---|
| $y = ax^2 + bx + c$ | y-int c; vertex $x = -b/2a$ |
| $y = a(x - h)^2 + k$ | Vertex (h, k) |
| $y = a(x - p)(x - q)$ | Zeros p & q |

**$a > 0$** opens up (min); **$a < 0$** opens down (max). Larger $|a| \rightarrow$ narrower, steeper parabola; a = stretch.

**Sum & product of roots**: sum = $-b/a$, product = $c/a$. Skip solving when asked for just these: $x^2 - 5x + 6 \rightarrow 5, 6$.

**Completing the square**: $x^2 + 6x \rightarrow (x + 3)^2 - 9$; take half of b, square it, then add & subtract it. Gives vertex form.

**Max/min word problems**: height $h(t) = -16t^2 + \ldots \rightarrow$ vertex gives max height & the time it occurs.

**Vertex form from factors**: zeros at p & q → vertex $x = (p + q)/2$, the midpoint of the zeros (by symmetry).

⚠ In the formula, −b means the opposite of b. If $b = -6$, then $-b = +6$. Watch double negatives.

## Radicals & Rational Expressions

### Radicals

**Simplify**: pull out perfect squares: $\sqrt{72} = \sqrt{(36 \cdot 2)} = 6\sqrt{2}$. Rule: $\sqrt{(ab)} = \sqrt{a} \cdot \sqrt{b}$ (never split sums).

**Add** only like radicals: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$.

**Multiply** freely: $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$, $\sqrt{5} \cdot \sqrt{5} = 5$.

**Rationalize**: $6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$. Conjugate for binomial denominators: multiply by $(a - \sqrt{b})/(a - \sqrt{b})$.

**Fractional exponents**: $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$; $16^{3/4} = (\sqrt[4]{16})^3 = 8$. Numerator = power, denominator = root.

### Radical Equations

**Steps**: isolate the radical, square both sides, solve, then CHECK every answer in the original equation.

**Example**: $\sqrt{(x + 7)} = x - 5 \rightarrow x + 7 = x^2 - 10x + 25 \rightarrow x = 9$ or 2; only $x = 9$ works ($x = 2$ gives $3 = -3$).

**Extraneous solution**: value from squaring that fails the original (makes a radical equal a negative).

**Cube roots**: $\sqrt[3]{(-8)} = -2$ (negatives allowed); cubing both sides never creates extraneous roots.

### Rational Expressions & Equations

| Task | Move |
|---|---|
| Simplify | Factor, cancel factors |
| Multiply | Factor, cancel, multiply |
| Divide | Flip 2nd, multiply |
| Add/subtract | Common denominator |

**Solve**: multiply both sides by the LCD; check answers that make any denominator 0 (excluded values, not solutions).

**Cancel factors, never terms**: $(x + 2)/(x + 5)$ doesn't reduce; $(x + 2)(x - 1)/(x + 2) = x - 1$, $x \neq -2$ (still excluded).

⚠ Always check radical & rational solutions. The ACT lists the extraneous answers as tempting choices.

✓ Decimal answer choices? Enter $\sqrt{72}$ & $6\sqrt{2}$ on your calculator to confirm they match (both ≈ 8.485).

## Exponential Equations & Modeling

### Exponential Models

**Growth/decay**: $y = a(1 \pm r)^t$. a = start, r = % rate as a decimal, t = time periods (years, months, etc.).

| Situation | Model |
|---|---|
| Grows 5%/yr | $a(1.05)^t$ |
| Shrinks 20%/yr | $a(0.80)^t$ |
| Doubles every 3 yr | $a \cdot 2^{t/3}$ |
| Halves every 10 yr | $a(\frac{1}{2})^{t/10}$ |

**Base b**: $b > 1 \rightarrow$ growth; $0 < b < 1 \rightarrow$ decay (nears 0, never reaches it). y-int = a, the value at $t = 0$.

**Compound interest**: $A = P(1 + r/n)^{nt}$; n = times compounded per year (monthly → n = 12, quarterly → n = 4).

### Solving Exponential Equations

**Same base**: $3^{x+1} = 81 = 3^4 \rightarrow x + 1 = 4 \rightarrow x = 3$. Rewrite both sides as powers of 1 common base first.

**Different bases**: use the calculator (graph & intersect) or logs: $x = \log(c)/\log(b)$ for $b^x = c$. $2^x = 10 \rightarrow x \approx 3.32$.

**Percent change**: (new − old)/old × 100; two 10% raises = $\times 1.1^2 = 1.21$, a 21% total increase, not 20%.

### Linear vs. Exponential

**Linear**: adds the same amount each step. **Exponential**: multiplies by the same factor each step.

**Translating words**: "is" → =, "of" → ×, "per" → ÷, "more than" → +, "less than" reverses order.

**"5 less than x"** = $x - 5$, not $5 - x$. "x decreased by 30%" = 0.7x, not $x - 0.3$. "Twice the sum" = $2(x + y)$.

⚠ A 20% decrease uses 0.80 as the base, not 0.20. Base = 1 − rate for decay, 1 + rate for growth.

✓ Table test: constant differences → linear; constant ratios → exponential. Check 3 rows to be sure.