The Simple ACT® Enhanced Math Functions Cheat Sheet

Function Basics & Notation

What a Function Is

Function: each input (x) gets exactly 1 output (y); outputs can repeat, but inputs can't.

Vertical line test: if any vertical line hits the graph twice, it isn't a function of x at all.

Evaluating

f(3): replace every x w/ 3. For $f(x) = 2x^2 - x$, $f(3) = 2(9) - 3 = 18 - 3 = 15$. Do powers first.

Negatives: use parentheses. For $f(x) = x^2 - 4x$, $f(-2) = (-2)^2 - 4(-2) = 4 + 8 = 12$.

Expression inputs: $f(a + 1)$ for $f(x) = x^2$ is $(a + 1)^2 = a^2 + 2a + 1$, not $a^2 + 1$ (a common trap).

Solve f(x) = k: set the rule equal to k: $3x - 7 = 11 \rightarrow 3x = 18 \rightarrow x = 6$, so $f(6) = 11$. x is the answer.

Notation	Means
f(2) = 5	Point (2, 5) on graph
f(x) = 0	x-intercepts (zeros)
f(0)	y-intercept
f(a) > g(a)	f above g at x = a

Reading Graphs

Graph: f(4) is the height of the graph at x = 4; f(x) = 0 at the x-intercepts (the zeros).

Increasing: the graph rises from left to right; **decreasing**: it falls from left to right.

⚠ f(x) means "f of x," not f times x.

✓ Stuck? Plug in easy numbers like x = 2 & test each answer choice to see which matches.

Domain & Range

Definitions

Domain: all allowed inputs (x-values). **Range**: all possible outputs (y-values) it gives.

Polynomials: lines & parabolas have a domain of all real numbers unless the context limits it.

Restrictions to Check

Function has	Rule	Example
Denominator	≠ 0	$1/(x - 3)$: $x \neq 3$
Even root	Inside ≥ 0	$\sqrt{(x + 2)}$: $x \geq -2$
Log	Inside > 0	$\log(x - 1)$: $x > 1$
Context	Makes sense	Time $t \geq 0$

Combine: $\sqrt{x}/(x - 4)$ needs $x \geq 0$ AND $x \neq 4$.

Log inputs: $\log(x - 1)$ needs $x - 1 > 0$, so $x > 1$.

Finding the Range

Parabola $y = a(x - h)^2 + k$: range $y \geq k$ if $a > 0$; range $y \leq k$ if $a < 0$ (set by the vertex).

Square root: $y = \sqrt{x}$ has range $y \geq 0$, since a square root never gives a negative output.

From a graph: domain = how far it spreads left to right; range = how far bottom to top.

Exponential: $y = 2^x$ has range $y > 0$, since its outputs are always positive numbers.

Notation

Interval notation: [ ] includes the endpoint; ( ) excludes it. $x \geq -2$ is written $[-2, \infty)$.

⚠ Domain = x-values; range = y-values. Swapping them is a common wrong answer.

✓ Graph it on a calculator & trace the edges to check the domain & range quickly.

Transformations

Shifts

Equation	Effect on f(x)
f(x) + k	Up k
f(x) − k	Down k
f(x − h)	Right h
f(x + h)	Left h

Inside the parentheses: moves the graph left or right, opposite to the sign: f(x − 2) goes right 2.

Outside f: moves the graph up or down, the same way as the sign: f(x) + 3 goes up 3.

Reflections & Stretches

Equation	Effect
−f(x)	Reflect over x-axis
f(−x)	Reflect over y-axis
a · f(x), a > 1	Vertical stretch
a · f(x), 0 < a < 1	Vertical shrink
f(bx), b > 1	Horizontal shrink

Applying Them

Example: $y = -(x - 2)^2 + 5$ flips $y = x^2$ upside down & moves it right 2 & up 5, to vertex (2, 5).

Point tracking: if (3, 4) is on f, then (5, 3) is on f(x − 2) − 1: move right 2 & down 1.

Absolute value: $y = |x - h| + k$ is a V-shape w/ its vertex at (h, k); a minus sign in front flips it.

⚠ f(x + 3) moves LEFT 3, not right.

✓ Check 1 point: see where the original vertex lands after you apply the changes.

Linear & Quadratic Functions

Linear Functions

f(x) = mx + b: m is the constant rate of change & b is the y-intercept, the value at x = 0.

From 2 points: (1, 5) & (4, 14) → slope = 9/3 = 3 → $5 = 3(1) + b \rightarrow b = 2$, so $f(x) = 3x + 2$.

Quadratic Forms

Form	Shows
$ax^2 + bx + c$	y-int c; vertex $x = -b/2a$
$a(x - h)^2 + k$	Vertex (h, k)
$a(x - p)(x - q)$	Zeros p & q

Vertex: $x = -b/(2a)$. For $y = x^2 - 6x + 5$, $x = 3$ & $y = 9 - 18 + 5 = -4$, so the vertex is (3, −4).

Max or min: opens up ($a > 0$) → minimum at the vertex; opens down ($a < 0$) → maximum there.

Axis of symmetry: x = h; the 2 zeros sit equally far on each side, so h is their average.

Zeros

Zeros: factor, use the quadratic formula or read the x-intercepts from the graph.

Discriminant $b^2 - 4ac$: positive → 2 real zeros; 0 → 1 zero; negative → no real zeros.

Projectile: $h(t) = -16t^2 + 64t$ peaks at $t = 2$ s (64 ft high) & lands at $t = 4$ s, when $h = 0$.

Sum of zeros = −b/a & product of zeros = c/a; use these when you don't need each zero.

⚠ $y = (x + 3)^2 - 1$ has vertex (−3, −1), not (3, −1).

✓ Use the calculator's max, min & zero tools to check your answers in seconds.

Polynomial & Rational Functions

Polynomial End Behavior

Degree	Leading coeff.	Ends
Even	+	Both up
Even	−	Both down
Odd	+	Down left, up right
Odd	−	Up left, down right

Degree n: up to n real zeros & up to n − 1 turns.

Zeros & Multiplicity

From factors: $(x - 1)(x + 2)^2$ has zeros at $x = 1$ & $x = -2$. The signs flip from the factors.

Multiplicity: odd power → crosses the x-axis; even power → touches & bounces back.

Factor & zero: if f(k) = 0, then (x − k) is a factor of f(x), & x = k is an x-intercept of it.

Rational Functions

Vertical asymptote: where the bottom = 0 after canceling, like x = 3 for $1/(x - 3)$.

Hole: a factor that cancels from top & bottom makes a hole, not an asymptote.

Degrees	Horizontal asymptote
Top < bottom	y = 0
Top = bottom	y = ratio of leading coeffs.
Top > bottom	None

⚠ Cancel common factors first to tell holes apart from vertical asymptotes on a graph.

✓ Check the ends first: the degree & leading sign often rule out 2 answer choices.

Exponential & Logarithmic Functions

Exponential Functions

$f(x) = a \cdot b^x$: a = starting value; b > 1 means growth, 0 < b < 1 means decay.

Asymptote: the graph gets closer & closer to y = 0 but never touches or crosses it.

Percent: 8% growth → b = 1.08; 8% decay → b = 0.92 (not 0.08). Base = 1 ± rate.

Logarithms

Definition: $\log_2 32 = 5$ b/c $2^5 = 32$. The value of a log is always an exponent of the base.

Rule	Statement
Product	$\log(xy) = \log x + \log y$
Quotient	$\log(x/y) = \log x - \log y$
Power	$\log(x^n) = n \log x$
Change of base	$\log_b x = \log x / \log b$

Log graph: $y = \log x$ is defined only for $x > 0$ & always passes through the point (1, 0).

Logs & exponents: $y = \log_b x$ is the inverse of $y = b^x$, so the graphs mirror over y = x.

Solving Equations

Same base: $9^x = 27 \rightarrow 3^{2x} = 3^3 \rightarrow x = 3/2$.

Rewrite logs: $\log_3(x + 1) = 2 \rightarrow x + 1 = 9 \rightarrow x = 8$.

Special values: $\log_b 1 = 0$ & $\log_b b = 1$.

⚠ $\log(x + y) \neq \log x + \log y$. The log rules only work for products, quotients & powers.

✓ Equal ratios in a table → exponential, not linear.

Composition, Inverses & Piecewise

Composition

f(g(x)): work inside out; find g(x) first, then put that output into f as its new input.

Example: $f(x) = x^2 + 1$ & $g(x) = 2x - 3 \rightarrow g(2) = 1$, so $f(g(2)) = f(1) = 1 + 1 = 2$ at the end.

Order matters: g(f(2)) = g(5) = 7, a different answer than f(g(2)) = 2. Read the order carefully.

Inverse Functions

Find the inverse: swap x & y, then solve for y: $f(x) = 3x - 6$ gives $f^{-1}(x) = (x + 6)/3$.

Key fact: if f(a) = b, then $f^{-1}(b) = a$; the 2 graphs are reflections over the line y = x.

One-to-one: only functions that pass the horizontal line test have inverses.

Piecewise Functions

x-value	Use rule
$x < 2$	$f(x) = x + 4$
$x \geq 2$	$f(x) = x^2$

Evaluate: pick the rule whose interval contains x: here $f(1) = 1 + 4 = 5$ & $f(3) = 3^2 = 9$.

Absolute value: $|x - 3| = 5 \rightarrow x - 3 = 5$ or $x - 3 = -5$, so $x = 8$ or $x = -2$ (2 answers).

Boundaries: check ≤ vs. <; a closed dot on a graph includes the endpoint, an open dot doesn't.

⚠ $f^{-1}(x)$ is the inverse, not $1/f(x)$.

✓ Composition from tables: do 2 lookups, starting w/ the inside function.

Trig Functions & Test Strategy

Sine & Cosine Graphs

$y = a \sin(bx) + d$: amplitude $|a|$, period $2\pi/|b|$ & midline y = d. Cosine works the same way.

Example: $y = 3 \sin(2x) + 1$ has amplitude 3, period π, midline y = 1 & range [−2, 4].

Function	Starts at x = 0	Range (a = 1)
sin x	0, rising	[−1, 1]
cos x	1, max	[−1, 1]
tan x	0, asymptotes	All reals

Unit Circle Values

θ	0	π/6	π/4	π/3	π/2
sin	0	1/2	√2/2	√3/2	1
cos	1	√3/2	√2/2	1/2	0

Radians: π radians = 180°, so 60° = π/3.

Tangent: $\tan\theta = \sin\theta / \cos\theta$; it's undefined where $\cos\theta = 0$, such as at $\theta = \pi/2$ (90°).

Test Strategy

Format: 45 questions in 50 minutes, 4 answer choices each, w/ a calculator allowed throughout.

No formula sheet: memorize the vertex, asymptote & log rules; the ACT doesn't give them.

Graph it: a graphing calculator finds zeros, maxes & intersection points for you in seconds.

⚠ Check degree vs. radian mode on your calculator before evaluating any trig.

✓ No penalty for guessing, so answer every question before time runs out.