The Ultimate ACT® Enhanced Math Functions Cheat Sheet

Function Basics & Notation

What a Function Is

Function: a rule that gives each input (x) exactly 1 output (y). Inputs may repeat outputs, never the reverse.

Vertical line test: if any vertical line hits a graph more than once, the graph is NOT a function of x.

From a table: an x-value listed twice w/ different y-values → not a function. The same y twice is fine.

Evaluating

f(3): replace every x w/ 3. f(x) = 2x² − x → f(3) = 2(9) − 3 = 15. Use parentheses around negatives.

f(−2) for f(x) = x² − 4x → (−2)² − 4(−2) = 4 + 8 = 12. Forgetting parentheses gives −4 + 8, a trap.

Expressions as inputs: f(a + 1) for f(x) = x² → (a + 1)² = a² + 2a + 1, not a² + 1 (a very common trap).

Solve f(x) = k: set the rule equal to k & solve: 3x − 7 = 11 → 3x = 18 → x = 6, so f(6) = 11 (x is the answer).

Notation	Means
f(2) = 5	Point (2, 5) on graph
f(x) = 0	x-intercepts (zeros)
f(0)	y-intercept
f(a) > g(a)	f above g at x = a

Reading Graphs & Tables

Graph: f(4) is the height (y) of the graph at x = 4; f(x) = 2 means find every x where the height is 2.

Increasing: graph rises left → right; **decreasing**: falls; **max/min**: the highest/lowest point on it.

Average rate of change on [a, b] = [f(b) − f(a)] / (b − a), the slope of the line between the 2 points.

⚠ f(x) is output notation, not f times x. f(2 + 3) ≠ f(2) + f(3).

✓ Stuck on an abstract rule? Plug in easy numbers (x = 0, 1, 2) & test each answer choice to match.

Domain & Range

Definitions

Domain: all allowed inputs (x-values). **Range**: all possible outputs (y-values) the function produces.

Default: polynomials (lines, parabolas, cubics) have domain = all real numbers unless context limits it.

Restrictions to Check

Function has	Rule	Example
Denominator	≠ 0	1/(x − 3): x ≠ 3
Even root	Inside ≥ 0	√(x + 2): x ≥ −2
Log	Inside > 0	log(x − 1): x > 1
Context	Makes sense	Time t ≥ 0

Combine rules: √x/(x − 4) → x ≥ 0 AND x ≠ 4. Both restrictions apply at once, so x = 4 is excluded.

Odd roots: $\sqrt[3]{x}$ accepts negatives, so its domain is all reals.

Finding the Range

Parabola y = a(x − h)² + k: if a > 0, range y ≥ k; if a < 0, range y ≤ k. The vertex's y-value sets the limit.

Square root: y = √x has range y ≥ 0; y = −√x + 3 has y ≤ 3.

Exponential: y = 2^x has range y > 0 (never touches y = 0).

From a graph: domain = left-to-right extent; range = bottom-to-top extent of the curve, incl. arrows.

Notation

Interval notation: [ ] includes the endpoint, ( ) excludes it. x ≥ −2 → [−2, ∞); infinity always gets a ( ).

Open vs. closed dots: an open circle = endpoint not included; a closed (filled) dot = endpoint included.

⚠ Domain is about x; range is about y. Swapping them is a classic wrong answer on graph questions.

✓ Graph it on a calculator & trace the edges to check domain & range quickly before choosing.

Transformations

Shifts

Equation	Effect on f(x)
f(x) + k	Up k
f(x) − k	Down k
f(x − h)	Right h
f(x + h)	Left h

Inside = opposite: changes inside the parentheses move horizontally & go the opposite way of the sign.

Outside = expected: changes outside f affect y-values & move the way the sign says (+ up, − down).

Reflections & Stretches

Equation	Effect
−f(x)	Reflect over x-axis
f(−x)	Reflect over y-axis
a · f(x), a > 1	Vertical stretch
a · f(x), 0 < a < 1	Vertical shrink
f(bx), b > 1	Horizontal shrink

Applying Them

Example: y = −(x − 2)² + 5 is y = x² reflected over the x-axis, shifted right 2 & up 5; its vertex is (2, 5).

Point tracking: (3, 4) on f → (5, 3) on f(x − 2) − 1.

Order: apply stretches & reflections before shifts when building a graph from its parent function.

Parent functions: y = x, x², x³, |x|, √x, 1/x, 2^x, log x; know each basic shape & 1 or 2 key points on it.

Absolute value y = a|x − h| + k: a V-shape w/ vertex (h, k); a < 0 flips it to open downward; |a| sets steepness.

⚠ f(x + 3) moves LEFT 3. Inside changes work backward.

✓ Test 1 point: plug the original vertex into the new rule to confirm exactly where it lands on the new graph.

Linear & Quadratic Functions

Linear Functions

f(x) = mx + b: constant rate of change m; equal steps in x give equal changes in f(x), unlike curves.

Build from 2 points: (1, 5) & (4, 14) → m = 9/3 = 3 → 5 = 3(1) + b → b = 2 → f(x) = 3x + 2. Check (4, 14).

Quadratic Forms

Form	Shows
ax² + bx + c	y-int c; vertex x = −b/2a
a(x − h)² + k	Vertex (h, k)
a(x − p)(x − q)	Zeros p & q

Vertex: x = −b/(2a), then plug in for y. y = x² − 6x + 5 → x = 3, y = 9 − 18 + 5 = −4, so the vertex is (3, −4).

Axis of symmetry: x = h; the zeros sit equally far on either side of it, so h = (p + q)/2, their average.

Max or min: a > 0 → opens up, minimum at vertex; a < 0 → opens down, maximum at vertex (the k-value).

Zeros & Discriminant

Zeros: factor, use the quadratic formula x = [−b ± √(b² − 4ac)]/2a, or read x-intercepts from a graph.

Discriminant b² − 4ac: > 0 → 2 real zeros; = 0 → 1 (touches the x-axis); < 0 → no real zeros (2 complex).

Sum & product of zeros: sum = −b/a, product = c/a; handy when you don't need each zero by itself.

Projectile: h(t) = −16t² + v_0t + h_0 (feet); max height at the vertex; the object lands when h(t) = 0.

Example: h(t) = −16t² + 64t → vertex t = 2 s, h(2) = 64 ft; lands at t = 4 s (the other zero is t = 0).

⚠ Vertex form a(x − h)² + k has vertex (h, k): y = (x + 3)² − 1 → vertex (−3, −1), not (3, −1). Mind signs.

✓ Graph the quadratic & use the calculator's max/min & zero features to check answers in seconds.

Polynomial & Rational Functions

Polynomial End Behavior

Degree	Leading coeff.	Ends
Even	+	Both up
Even	−	Both down
Odd	+	Down left, up right
Odd	−	Up left, down right

Degree n: at most n real zeros & n − 1 turns.

Zeros & Multiplicity

From factors: f(x) = (x − 1)(x + 2)² → zeros x = 1 & x = −2; read each factor's sign carefully (x + 2 → −2).

Multiplicity: odd power → graph crosses the x-axis; even power → graph touches & turns back.

Write from zeros: zeros 2 & −3 → f(x) = a(x − 2)(x + 3); use a given point on the graph to solve for a.

Remainder theorem: f(x) ÷ (x − k) has remainder f(k); f(k) = 0 means (x − k) is a factor of f(x), & k is a zero.

Rational Functions

Vertical asymptote: denominator = 0 after canceling common factors. 1/(x − 3) → x = 3 is the asymptote.

Hole: a factor cancels from top & bottom. (x − 2)(x + 1)/(x − 2) → a hole at x = 2, not an asymptote there.

Degrees	Horizontal asymptote
Top < bottom	y = 0
Top = bottom	y = ratio of leading coeffs.
Top > bottom	None

Example: (2x + 1)/(x − 4) → vertical asymptote x = 4 & horizontal asymptote y = 2/1 = 2 (equal degrees).

⚠ Cancel common factors first. A factor that cancels is a hole in the graph, not a vertical asymptote.

✓ Sketch the ends first: degree & leading sign alone often eliminate 2 or 3 of the answer choices.

Exponential & Logarithmic Functions

Exponential Functions

f(x) = a · b^x: a = initial value (the y-intercept); b > 1 → growth, 0 < b < 1 → decay; b must be positive & ≠ 1.

Asymptote: horizontal at y = 0 (or y = k for a · b^x + k); domain all reals, range y > 0 if a > 0 & k = 0.

Percent models: grows r% → b = 1 + r; decays r% → b = 1 − r. An 8% decay → b = 0.92, not 0.08 or 1.08.

Doubling/half-life: $P_0 \cdot 2^{t/d}$ doubles each d; $P_0(0.5)^{t/h}$ halves each h; keep d, h & t in the same units.

Logarithms

Definition: $\log_b c = a \Leftrightarrow b^a = c$, so $\log_2 32 = 5$ b/c $2^5 = 32$.

Rule	Statement
Product	log(xy) = log x + log y
Quotient	log(x/y) = log x − log y
Power	$\log(x^n) = n \log x$
Change of base	$\log_b x = \log x / \log b$

Special values: $\log_b 1 = 0$ & $\log_b b = 1$.

Log graph: $y = \log_b x$ is the inverse of b^x; domain x > 0, vertical asymptote x = 0, passes (1, 0).

Solving Equations

Same base: $9^x = 27 \rightarrow 3^{2x} = 3^3 \rightarrow 2x = 3 \rightarrow x = 3/2$. Rewrite both sides as powers of 1 common base first.

Rewrite logs: $\log_3(x + 1) = 2 \rightarrow x + 1 = 3^2 = 9 \rightarrow x = 8$; check that the log's argument stays positive.

⚠ log(x + y) ≠ log x + log y. Log rules work only for products, quotients & powers inside.

✓ Exponential vs. linear? Equal ratios in a table → exponential; equal differences → linear.

Composition, Inverses & Piecewise

Composition

f(g(x)): work inside out: find g(x), then plug that output into f. It's also written (f ∘ g)(x) on the ACT test.

Example: f(x) = x² + 1, g(x) = 2x − 3 → f(g(2)) = f(1) = 2; g(f(2)) = g(5) = 7. The order really matters.

Algebraic: f(g(x)) = (2x − 3)² + 1 = 4x² − 12x + 10. Substitute the whole expression, then expand it fully.

From tables: to find f(g(4)), read g(4) in the g table, then look up that output as the input in the f table.

Inverse Functions

Inverse f^{-1} undoes f: swap x & y, then solve for y. f(x) = 3x − 6 → $f^{-1}(x)$ = (x + 6)/3 = x/3 + 2, a line too.

Key facts: if f(a) = b, then $f^{-1}(b) = a$; the graphs reflect over y = x; the domain & range swap places.

Check: $f(f^{-1}(x)) = x$. Only one-to-one functions (passing the horizontal line test) have inverse functions.

Piecewise Functions

x-value	Use rule
x < 2	f(x) = x + 4
x ≥ 2	f(x) = x²

Evaluate: find which interval contains x, then use only that rule: here f(1) = 5, f(2) = 4 & f(3) = 9 (x ≥ 2 rule).

Boundary: check ≤ vs. < carefully; the closed dot on the graph shows which rule includes the endpoint.

Absolute value as piecewise: |x| = x for x ≥ 0 & −x for x < 0. So |x − 3| = 5 gives x = 8 or x = −2 (2 answers).

Step functions: output jumps at set inputs (shipping costs, parking fees); read the correct step carefully.

⚠ $f^{-1}(x)$ means the inverse, NOT 1/f(x).

✓ Composition from a graph: read g(a) on g's graph, then find f at that x-value on f's graph. 2 lookups.

Trig Functions & Test Strategy

Sine & Cosine Graphs

y = a sin(bx) + d: amplitude |a|, period 2π/|b|, midline y = d.

Example: y = 3 sin(2x) + 1 → amplitude 3, period π, midline y = 1, range [−2, 4], so its max value is 4.

Function	Starts at x = 0	Range (a = 1)
sin x	0, rising	[−1, 1]
cos x	1, max	[−1, 1]
tan x	0, asymptotes	All reals

Unit Circle Values

θ	0	π/6	π/4	π/3	π/2
sin	0	1/2	√2/2	√3/2	1
cos	1	√3/2	√2/2	1/2	0

Radians: π radians = 180°, so 60° = π/3 & 135° = 3π/4. Check your calculator's mode before evaluating trig.

Signs by quadrant: All (I), Sine (II), Tangent (III), Cosine (IV) are positive: "All Students Take Calculus."

Tangent: tan θ = sin θ / cos θ; undefined where cos θ = 0 (θ = π/2), which gives vertical asymptotes.

Test Strategy

Format: enhanced ACT math has 45 questions in 50 minutes, 4 answer choices, calculator allowed throughout.

No formula sheet: memorize vertex, asymptote, log & transformation rules; they won't be given.

Graph it: a graphing calculator answers many function questions directly (zeros, max, intersections).

Backsolve & plug in: test answer choices or plug simple numbers into rules when algebra is slow.

Pacing: about 1 minute per question; skip a long function problem & return to it after the easier ones.

⚠ Degree vs. radian mode errors cost easy points. Match the calculator mode to the question's angle units.

✓ No penalty for guessing: answer every question, & flag the hard ones to come back to if time allows.