# The Simple ACT® Enhanced Math Geometry Cheat Sheet



## Lines, Angles & Triangles

### Angle Basics

**Complementary** → sum 90°; **supplementary** → 180°; around a point → 360° total.

**Vertical ∠s** (opposite at an X of 2 lines) are always =; neighbors at the X sum to 180°.

### Parallel Lines

| Angle pair | Relationship |
|---|---|
| Corresponding | Equal |
| Alternate interior | Equal |
| Same-side interior | Add to 180° |

**Shortcut**: ∥ lines → every ∠ = x or 180° − x.

### Triangle Rules

**∠ sum** = 180°: 50° & 65° → 3rd ∠ = 65°.

**Exterior ∠** = sum of the 2 far interior ∠s: 40° & 75° → the exterior ∠ = 115° (40 + 75).

**Triangle inequality**: each side < sum of the other 2; sides 4 & 9 → 5 < 3rd side < 13.

**Side ↔ ∠**: longest side opp. largest ∠.

**Isosceles**: 2 = sides ↔ 2 = base ∠s; top ∠ 40° → each base ∠ = (180 − 40)/2 = 70°.

**Equilateral**: 3 = sides, all ∠s 60°; height = s√3/2.

**Isosceles right △** = 45-45-90.

**Perimeter** = add sides: 5 + 12 + 13 = 30.

**Area** = ½bh w/ h ⊥ to the base.

**Midsegment**: ∥ to the 3rd side & = ½ its length.

**Centroid**: medians meet ⅔ from each vertex.

⚠ Not to scale → trust the numbers.

✓ Mark each ∠ you find on the figure.

## Right Triangles & Special Triangles

### Pythagorean Theorem

**$a^2 + b^2 = c^2$**, c = hyp (longest, opp. 90°).

**Missing leg**: hyp 13, leg 5 → √(169 − 25) = √144 = 12; subtract squares to find any leg.

| Triple | Multiples |
|---|---|
| 3-4-5 | 6-8-10, 9-12-15 |
| 5-12-13 | 10-24-26 |
| 8-15-17 | 16-30-34 |
| 7-24-25 | 14-48-50 |

### Special Right Triangles

| Triangle | Side ratio |
|---|---|
| 45°-45°-90° | x : x : x√2 |
| 30°-60°-90° | x : x√3 : 2x |

**45-45-90**: leg 5 → hyp 5√2; diag. = s√2.

**30-60-90**: short leg 4 → long leg 4√3, hyp 8 (2 × short); the short leg is opp. 30°.

**Converse** (c = longest): $a^2 + b^2 = c^2$ → right △; $a^2 + b^2 > c^2$ → acute; $< c^2$ → obtuse.

**Equilateral height** = (s/2)√3: side 10 → 5√3.

### Where They Show Up

**Rectangle diagonal**: 6 × 8 → √(36 + 64) = 10. **Box diagonal** = $\sqrt{l^2 + w^2 + h^2}$ for a 3D box.

**Cube diagonals**: face s√2; space s√3.

**Distance** = $\sqrt{(\Delta x^2 + \Delta y^2)}$: Pythagorean thm.

**Ladder**: 10 ft, base 6 ft out → 8 ft up.

⚠ c = hyp; finding a leg → subtract.

✓ Spot a triple before using a calculator.

## Congruence & Similarity

### Congruent Triangles

**Congruent (≅)**: same shape & size → all matching sides & ∠s are =; order of letters matters.

| Test | What matches |
|---|---|
| SSS | 3 sides |
| SAS | 2 sides + angle between |
| ASA / AAS | 2 angles + a side |
| HL | Right △: hyp + leg |

**Not tests**: AAA → similar only; SSA does NOT prove ≅ (HL in right △s is the exception).

### Similar Triangles

**Similar (~)**: = ∠s & proportional sides; **AA** (2 pairs of = ∠s) is enough to prove similarity.

**Scale factor**: sides 4 & 10 match → k = 2.5; side 6 → 6 × 2.5 = 15. Match sides by their ∠s.

**Shadows**: 6 ft person → 4 ft shadow; tree's shadow 20 ft → tree = 6·20/4 = 30 ft.

**Side-splitter**: a line ∥ to 1 side cuts the other 2 sides in proportion (the same ratio).

### Scaling

| If length × k | Then |
|---|---|
| Perimeter | × k |
| Area | × $k^2$ |
| Volume | × $k^3$ |

**Ex**: sides × 2 → area × 4 & volume × 8.

⚠ Match sides by ∠s, not by spot in the drawing.

✓ Set up small/big = small/big.

## Right Triangle Trig & the Unit Circle

### SOH-CAH-TOA

| Ratio | Definition | 3-4-5 △ |
|---|---|---|
| sin θ | opp / hyp | 3/5 |
| cos θ | adj / hyp | 4/5 |
| tan θ | opp / adj | 3/4 |

**Find a side**: 35°, hyp 20 → opp = 20 sin 35° ≈ 11.5.

**Find an ∠**: opp 7, adj 10 → θ = $\tan^{-1}(7/10)$ ≈ 35° (set the calculator to degree mode).

### Unit Circle Values

| θ | rad | sin | cos |
|---|---|---|---|
| 0° | 0 | 0 | 1 |
| 30° | π/6 | ½ | √3/2 |
| 45° | π/4 | √2/2 | √2/2 |
| 60° | π/3 | √3/2 | ½ |
| 90° | π/2 | 1 | 0 |

**Identities**: sin θ = cos(90° − θ); $\sin^2\theta + \cos^2\theta = 1$; tan θ = sin θ ÷ cos θ (3/5 ÷ 4/5 = 3/4).

### Non-Right Triangles

**Law of Sines**: a/sin A = b/sin B → AAS, ASA.

**Law of Cosines**: $c^2 = a^2 + b^2 - 2ab \cos C$ → use w/ 3 sides or 2 sides + the ∠ between.

**Area** = ½ab sin C (2 sides & included ∠).

**SSA ambiguous case**: 2 sides + a non-included ∠ can give 0, 1 or 2 different △s; check both.

**Elevation**: 50 ft away, 40° → h = 50 tan 40°.

⚠ Label opp & adj from the ∠ you use.

✓ Draw & label opp, adj, hyp first.

## Polygons & Area

### Area Formulas

| Shape | Area |
|---|---|
| Rectangle | length × width |
| Square | $side^2$ |
| Parallelogram | base × height |
| Trapezoid | $\frac{1}{2}(b_1 + b_2) \times h$ |
| Rhombus | $\frac{1}{2}d_1d_2$ |

**Trapezoid**: bases 6 & 10, h 4 → ½(6 + 10)(4) = 32.

**Height** = ⊥ distance, not a slanted side.

### Polygon Angles & Diagonals

**Interior sum** = (n − 2)·180°: pentagon 540°, hexagon 720°, octagon 1,080° (n = 8).

**Regular polygon**: each ∠ = sum ÷ n; hexagon → 720° ÷ 6 = 120°; exterior = 360°/n = 60°.

**Exterior ∠s** of any polygon sum to 360°.

**Diagonals** = n(n − 3)/2: hexagon → 9.

### Quadrilaterals

**Parallelogram**: opp. sides ∥ & =; opp. ∠s =; diagonals bisect each other (cut in half).

**Square** = rectangle + rhombus.

**Rhombus**: diagonals ⊥; A = $\frac{1}{2}d_1d_2$.

**Max area** for a fixed perimeter → square.

### Shaded Regions

**Shaded** = whole − unshaded: square 10 w/ circle r = 5 → 100 − 25π ≈ 21.5 square units.

⚠ Perimeter = length (units); area = square units.

✓ Split odd shapes into rectangles & △s.

## Circles

### Core Formulas

| Measure | Formula | r = 6 |
|---|---|---|
| Circumference | 2πr | 12π |
| Area | $\pi r^2$ | 36π |
| Arc (60°) | (θ/360)·2πr | 2π |
| Sector (60°) | (θ/360)·$\pi r^2$ | 6π |

**Reverse**: area 49π → r = 7 → circumference 14π.

**Radians**: 180° = π → 90° = π/2, 60° = π/3, 45° = π/4; degrees × π/180 → radians.

### Angles, Arcs & Chords

**Central ∠** = its arc: 70° ∠ → 70° arc.

**Inscribed ∠** = ½ its arc: 80° arc → 40° ∠; an ∠ on a diameter (a semicircle) is always 90°.

**Same arc** → inscribed ∠s are equal.

**Tangent ⊥ radius** → right △ to solve.

**2 tangents** from 1 outside pt are =.

**Circle ↔ square**: circle in a square → side = diameter; square in a circle → diagonal = diameter.

**Chord**: radius ⊥ chord bisects it; chord 16, r = 10 → distance to center = √(100 − 64) = 6.

### Equation of a Circle

**$(x - h)^2 + (y - k)^2 = r^2$**: center (h, k), radius r; signs flip: $(x + 2)^2$ → h = −2, $(y - 3)^2$ → k = 3.

**Ex**: $(x + 2)^2 + (y - 3)^2 = 25$ → r = 5.

**Point test**: = $r^2$ on; < $r^2$ in; > $r^2$ out.

⚠ The equation shows $r^2$: $r^2 = 16$ → r = 4, not 16.

✓ Arc & sector = θ/360 of the whole circumference or the whole area.

## Solids: Volume & Surface Area

### Volume Formulas

| Solid | Volume |
|---|---|
| Box | length × width × height |
| Cylinder | $\pi r^2 h$ |
| Cone | $\frac{1}{3}\pi r^2 h$ |
| Pyramid | ⅓ × base area × h |
| Sphere | $(4/3)\pi r^3$ |

**Ex**: cylinder r 3, h 4 → $\pi(3^2)(4) = 36\pi$ units³.

**⅓ rule**: cone = ⅓ cylinder; pyramid = ⅓ prism.

### Surface Area

**Box**: SA = 2(lw + lh + wh): 2 × 3 × 4 → 52 units².

**Cube**: V = $s^3$, SA = $6s^2$; V 64 → s 4 → SA 96.

**Cylinder**: SA = $2\pi r^2 + 2\pi rh$ (2 circle ends + the side): r = 3, h = 4 → 18π + 24π = 42π.

**Sphere**: SA = $4\pi r^2$; r = 3 → 36π.

**Cone**: SA = $\pi r^2 + \pi r\ell$, ℓ = slant height.

### Problem Types

**Fill rate**: 120 ft³ tank at 2 ft³/min → 60 min.

**Scaling**: lengths × 2 → SA × 4, volume × 8.

**Units**: 1 ft³ = 12³ = 1,728 in³ (cube the conversion).

**Displacement**: water rise = object volume ÷ the container's base area ($\pi r^2$ or lw).

**Melt & recast**: volume stays the same → set $V_{old} = V_{new}$ & solve for the new length.

**3D diagonal**: box $\sqrt{(l^2 + w^2 + h^2)}$; cube s√3.

⚠ Cone & sphere formulas aren't given: memorize them before test day.

✓ Units² vs. units³ rules out choices.

## Coordinate Geometry & Test Strategy

### Key Formulas

| Tool | (1, 2) & (7, 10) |
|---|---|
| Distance | √(36 + 64) = 10 |
| Midpoint | (4, 6) |
| Slope | 8/6 = 4/3 |

**Midpoint**: average the x's & the y's: ((1 + 7)/2, (2 + 10)/2) = (4, 6), the center of the segment.

**Endpoint**: midpoint (4, 6), end (1, 2) → other end = (2·4 − 1, 2·6 − 2) = (7, 10): double & subtract.

**Slope** = rise/run: (10 − 2)/(7 − 1) = 4/3.

∥ → = slopes; ⊥ → negative reciprocals (2 → −½).

### Transformations

| Move | Rule for (x, y) |
|---|---|
| Reflect over x-axis | (x, −y) |
| Reflect over y-axis | (−x, y) |
| Rotate 180° | (−x, −y) |
| Rotate 90° CCW | (−y, x) |

**Translation**: right 3, up 2 → (x + 3, y + 2).

**Dilation** × k → lengths × k, area × $k^2$.

### Test Strategy

**Format**: 45 Qs in 50 min, 4 choices, calculator OK.

**No formula sheet**: memorize formulas.

**Draw it**: sketch & label any figure given in words.

**Pacing**: ~1 min/Q; skip long ones, come back later.

⚠ ⊥ → negative reciprocal: 2/5 → −5/2.

✓ No guessing penalty → answer every question.