The Ultimate ACT® Enhanced Math Geometry Cheat Sheet

Lines, Angles & Triangles

Angle Basics

Complementary → sum 90°; **supplementary** → 180°; around a point → 360°; a linear pair is supplementary.

Vertical ∠s (opposite at an X of 2 lines) are always =; adjacent ∠s at the same X sum to 180°.

Parallel Lines + Transversal

Angle pair	Relationship
Corresponding	Equal
Alternate interior	Equal
Alternate exterior	Equal
Same-side interior	Add to 180°

Shortcut: ∥ lines → every ∠ = x or 180° − x.

Triangle Rules

∠ sum = 180°: 50° & 65° → 3rd = 65°. **Exterior ∠** = sum of 2 remote interior ∠s (40° + 75° → 115°).

Triangle inequality: each side < sum of the other 2. Sides 4 & 9 → 5 < 3rd side < 13 (never = 5 or 13).

Side ↔ ∠: longest side opp. largest ∠.

Isosceles: 2 = sides ↔ 2 = base ∠s; vertex 40° → each base ∠ = (180 − 40)/2 = 70°; base 50° → vertex 80°.

Equilateral: all ∠s 60°; A = $s^2\sqrt{3}/4$ (s = 6 → $9\sqrt{3} \approx 15.6$); height = $s\sqrt{3}/2$ (s = 6 → height $3\sqrt{3} \approx 5.2$).

A = ½bh, h ⊥ b; obtuse △ → h may fall outside the △.

Midsegment: ∥ 3rd side & = ½ its length.

Centroid: medians meet ⅔ of the way from each vertex.

∠ bisector thm: an ∠ bisector splits the opp. side in the same ratio as the 2 adjacent sides (6 : 9 → 2 : 3).

⊥ bisector: every pt on it is equidistant from both endpoints.

⚠ Not to scale → trust given values.

✓ Mark each ∠ you find; most chains = 2–3 steps.

Right Triangles & Special Triangles

Pythagorean Theorem

$a^2 + b^2 = c^2$, c = hyp (opp. 90°, longest side). Missing leg: hyp 13, leg 5 → $\sqrt{(169 - 25)} = \sqrt{144} = 12$ units.

Triple	Multiples
3-4-5	6-8-10, 9-12-15, 12-16-20
5-12-13	10-24-26
8-15-17	16-30-34
7-24-25	14-48-50

Converse (c = longest side): $a^2 + b^2 = c^2$ → right △; $a^2 + b^2 > c^2$ → acute △; $a^2 + b^2 < c^2$ → obtuse △.

Special Right Triangles

Triangle	Side ratio	Example
45°-45°-90°	x : x : x√2	leg 5 → hyp 5√2
30°-60°-90°	x : x√3 : 2x	short 4 → 4√3, 8

45-45-90: leg = hyp/√2; square diag. = s√2.

30-60-90: short leg opp. 30°; long leg = short·√3; hyp = 2·short. Hyp 12 → short leg 6, long leg 6√3.

Equilateral altitude: splits it into two 30-60-90 △s → height = (s/2)√3; side 10 → height 5√3 ≈ 8.7.

Trig link: sin 30° = ½, sin 45° = √2/2, sin 60° = √3/2 all come straight from these 2 special right △s.

Where They Hide

Rectangle diagonal: 6 × 8 → √(36 + 64) = 10. **Box diagonal** = $\sqrt{(l^2 + w^2 + h^2)}$: 2, 3, 6 → √49 = 7 units.

Cube: face diag. s√2; space diag. s√3.

Distance = $\sqrt{(\Delta x^2 + \Delta y^2)}$ = Pythagorean thm on the grid.

Ladder: ladder = hyp; 10 ft ladder, base 6 ft out → top 8 ft up.

Ramps: rise & run = legs, ramp = hyp; 5 ft up over 12 ft of run → a 13 ft ramp (it's the 5-12-13 triple, scale 1).

⚠ c = hyp; finding a leg → subtract squares.

✓ Spot a triple or special ratio before you calc.

Congruence & Similarity

Congruent Triangles

Congruent (≅): same shape + size; all corresponding sides & ∠s =; letter order shows which parts match.

Test	What matches
SSS	3 sides
SAS	2 sides + included angle
ASA / AAS	2 angles + a side
HL	Right △: hyp + leg

Not tests: AAA → similar only; SSA × for congruence (one exception: HL works, but only in right △s).

CPCTC: once △s ≅ → all corresponding parts ≅.

Similar Triangles

Similar (~): = ∠s & proportional sides; **AA** is enough. Also SSS~ & SAS~ (sides in the same ratio k).

Scale factor k: AB = 4, DE = 10 → k = 2.5; BC = 6 → EF = 15.

Shadows: 6 ft → 4 ft shadow; tree 20 ft → 30 ft.

Altitude to hyp: splits a right △ into 2 △s ~ the original; altitude² = product of the 2 hyp pieces (4 & 9 → 6).

Side-splitter: line ∥ 1 side → proportional.

Nested △s: a small △ inside a big one w/ a shared ∠ & ∥ sides → AA; set small/big = small/big.

Other ratios: perimeters, altitudes, medians & ∠ bisectors all scale by the same factor k as the sides do.

Scaling Area & Volume

If length × k	Then
Perimeter	× k
Area	× k^2
Volume	× k^3

Ex: sides × 2 → area × 4, volume × 8 (k^2 & k^3).

⚠ Match sides by ∠s, not by position in the figure.

✓ Area/vol ratio → take the length ratio first.

Right Triangle Trig & the Unit Circle

SOH-CAH-TOA

Ratio	Definition	For 3-4-5, θ opp 3
sin θ	opp / hyp	3/5
cos θ	adj / hyp	4/5
tan θ	opp / adj	3/4

Reciprocals: csc = hyp/opp; sec = hyp/adj.

Side: 35°, hyp 20 → opp = 20 sin 35° ≈ 11.5. **∠**: opp 7, adj 10 → θ = $\tan^{-1}(0.7) \approx 35°$ (degree mode).

Unit Circle Values

θ	sin	cos	tan
0°	0	1	0
30° (π/6)	½	√3/2	√3/3
45° (π/4)	√2/2	√2/2	1
60° (π/3)	√3/2	½	√3
90° (π/2)	1	0	undef.

ASTC: QI all +, QII sin +, QIII tan +, QIV cos +.

Ref. ∠: 150° → 30°; 225° → 45°; 300° → 60°; 390° → 30°.

Degrees ↔ radians: × π/180 or × 180/π; π/3 = 60°, 3π/4 = 135°, 5π/6 = 150°, 3π/2 = 270°, 2π = 360°.

Identities: sin θ = cos(90° − θ); $\sin^2\theta + \cos^2\theta = 1$; tan θ = sin θ/cos θ (sin 3/5 in QI → cos 4/5, tan 3/4).

Non-Right Triangles

Law of Sines: a/sin A = b/sin B → AAS, ASA, SSA.

SSA ambiguous case: given a, b & acute ∠A → 0, 1 or 2 △s; find B, then check if 180° − B also fits.

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$ → SSS or SAS.

Area = ½ab sin C (sides 8 & 10, 30° → 20).

Elevation: 50 ft away, 40° → h = 50 tan 40° ≈ 42 ft.

⚠ Opp/adj switch when you switch ∠s.

✓ Exact choices (20 sin 35°) → match setup.

Polygons & Area

Area Formulas

Shape	Area	Perimeter
Rectangle	lw	2l + 2w
Square	s^2	4s
Parallelogram	bh	2a + 2b
Trapezoid	$\frac{1}{2}(b_1 + b_2)h$	Add all sides
Rhombus	$\frac{1}{2}d_1d_2$ or bh	4s
Kite	$\frac{1}{2}d_1d_2$	Add all sides

Trapezoid: bases 6 & 10, h 4 → ½(6 + 10)(4) = 32; use the ⊥ height, never the slanted side's length (leg).

Polygon Angles & Diagonals

Interior sum = (n − 2)·180°: pentagon 540°, hexagon 720°, octagon 1,080°, decagon 1,440° (n = 10).

Regular n-gon: each interior = (n − 2)180°/n; each exterior = 360°/n (exterior ∠s always sum to 360°).

Find n: interior 140° → exterior 40° → n = 360/40 = 9.

Diagonals = n(n − 3)/2: hexagon → 6·3/2 = 9; octagon → 20.

Quadrilaterals

Parallelogram: opp. sides ∥ & =; opp. ∠s =.

Diagonals: ▱ → bisect each other; rectangle → =; rhombus → ⊥ & bisect ∠s; square → all of these at once.

Hierarchy: square ⊂ rectangle, rhombus ⊂ ▱.

Rhombus: diagonals 6 & 8 → A = ½(6)(8) = 24; side = $\sqrt{(3^2 + 4^2)} = 5$ b/c the diagonals ⊥ bisect each other.

Max area: for a fixed perimeter, a square gives the largest rectangle area (P = 20 → 5 × 5 = 25).

Composite & Shaded Regions

Shaded = whole − unshaded: square s = 10 w/ inscribed circle r = 5 → 100 − 25π ≈ 21.5 square units.

Composite: split L-shapes into rectangles.

⚠ Perimeter (units) ≠ area (units²); check what the Q asks.

✓ Regular hexagon = 6 equilateral △s → A = $3s^2\sqrt{3}/2$.

Circles

Core Formulas

Measure	Formula	r = 6
Circumference	2πr = πd	12π
Area	πr^2	36π
Arc length	(θ/360)·2πr	θ = 60° → 2π
Sector area	(θ/360)·πr^2	θ = 60° → 6π

Radians: arc s = rθ, sector area = $\frac{1}{2}r^2\theta$ (θ in radians).

Reverse: A = 49π → r = 7 → C = 14π (C = 2πr).

Angles, Arcs & Chords

Central ∠ = arc; **inscribed ∠** = ½ arc.

Same arc → inscribed ∠s are =; a quadrilateral inscribed in a circle → its opposite ∠s always sum to 180°.

Tangent ⊥ radius; 2 tangents from 1 pt are =.

Chord: radius ⊥ chord bisects it; chord 16, r = 10 → distance from center = $\sqrt{(100 - 64)} = 6$ units (½ chord = 8).

Intersecting chords: ∠ = ½(sum of the 2 intercepted arcs); chord pieces multiply equally: a·b = c·d.

Outside ∠ (2 secants or tangents) = ½(far arc − near arc).

Tangent–chord ∠ = ½ intercepted arc.

Tangent–secant: (tangent)² = whole secant × its outside piece; tangent 6, outside 4 → whole secant 9.

Circle ↔ square: circle in a square → side = diameter; square in a circle → diagonal = diameter.

Equation of a Circle

$(x - h)^2 + (y - k)^2 = r^2$: $(x + 2)^2 + (y - 3)^2 = 25$ → center (−2, 3), radius 5; signs flip inside the parentheses.

Complete the square: $x^2 + 6x + y^2 - 4y = 12$ → add 9 & 4 to both sides → $(x + 3)^2 + (y - 2)^2 = 25$ → r = 5.

Point check: plug in → = r^2 on; < r^2 inside; > r^2 outside.

⚠ The equation shows r^2: $r^2 = 16$ → r = 4.

✓ Arc/sector = θ/360 of C or A; simplify first.

Solids: Volume & Surface Area

Volume Formulas

Solid	Volume	r = 3, h = 4
Prism / box	Bh (or lwh)	Box 3×4×5: 60
Cylinder	$\pi r^2 h$	36π
Cone	$\frac{1}{3}\pi r^2 h$	12π
Pyramid	⅓Bh	Base area B
Sphere	$(4/3)\pi r^3$	36π
Hemisphere	$(2/3)\pi r^3$	18π

B = base area: triangular prism, B = 10, length 8 → V = 80.

⅓ rule: cone = ⅓ cylinder; pyramid = ⅓ prism.

Surface Area

Box: SA = 2(lw + lh + wh): 2 × 3 × 4 → 52. **Cube**: V = s^3, SA = $6s^2$ (V = 64 → s = 4 → SA = 96 units²).

Cylinder: SA = $2\pi r^2 + 2\pi rh$ (r 3, h 4 → 18π + 24π = 42π).

Sphere: SA = $4\pi r^2$ (r = 3 → 36π units²).

Cone: SA = $\pi r^2 + \pi r\ell$, slant $\ell = \sqrt{(r^2 + h^2)}$.

Cube from SA: SA 54 → face 9 → s = 3 → V = 27.

Melt & recast: total volume stays the same, so set V_{old} = V_{new} & then solve for the missing length.

Water depth: h = V ÷ πr^2; 50π in³ poured into a cylinder w/ r = 5 → the water is 50π/25π = 2 in deep.

Problem Types

Rate: 4 × 5 × 6 tank = 120 ft³ at 2 ft³/min → 60 min.

Displacement: water rise = object vol ÷ container base area.

Units: 1 ft³ = 1,728 in³; 1 yd³ = 27 ft³ (cube the length factor).

Scaling: dims × 2 → SA × 4, V × 8 (k^2, k^3).

3D diagonals: box $\sqrt{(l^2 + w^2 + h^2)}$; cube s√3; longest rod in a cylinder = $\sqrt{(d^2 + h^2)}$ (d = diameter, h = height).

⚠ Memorize $\frac{1}{3}\pi r^2 h$ & $(4/3)\pi r^3$: not given.

✓ units² vs. units³ rules out choices.

Coordinate Geometry & Test Strategy

Key Formulas

Tool	Formula	(1, 2) & (7, 10)
Distance	$\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$	$\sqrt{(36 + 64)} = 10$
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$	(4, 6)
Slope	$(y_2 - y_1)/(x_2 - x_1)$	8/6 = 4/3

∥ → = slopes; **⊥** → negative reciprocals.

Endpoint: midpoint (4, 6), one end (1, 2) → other end = (2·4 − 1, 2·6 − 2) = (7, 10): double it, then subtract.

Point-slope: $y - y_1 = m(x - x_1)$; **slope-intercept**: y = mx + b; collinear pts → equal slopes for every pair.

Transformations

Move	Rule for (x, y)
Reflect over x-axis	(x, −y)
Reflect over y-axis	(−x, y)
Reflect over y = x	(y, x)
Rotate 90° CCW	(−y, x)
Rotate 180°	(−x, −y)

Translation (x + a, y + b); **dilation** by k from origin → (kx, ky): lengths × k, area × k^2, ∠s unchanged.

Grid area: box it in, subtract corner △s.

Rotate 90° CW: (x, y) → (y, −x); 270° CCW = 90° CW.

Line of reflection = ⊥ bisector of a point & its image.

Test Strategy

Format: 45 Qs / 50 min, 4 choices, calculator.

No formula sheet: memorize every formula.

Draw it: sketch & label word problems.

Pacing: ~1 min/Q; skip long multi-step figures, return later.

⚠ ⊥ = negative RECIPROCAL: ⊥ to 2/5 is −5/2, not −2/5.

✓ No guessing penalty → answer every Q.