The Simple ACT® Enhanced Math Integrating Essential Skills Cheat Sheet

Integrating Essential Skills: Overview

What It Tests

IES: multistep problems that combine basic skills like rates, percents, area, volume & averages.

Weight: ≈ 20% of the enhanced ACT math section, about 8 of the 45 questions on the test.

Format: 45 questions in 50 minutes, 4 choices each, calculator allowed, no formula sheet.

Skill	Typical task
Percents	Discounts, tax, % change
Rates	Unit price, speed, work
Proportions	Scale, recipes, maps
Area/volume	Composite shapes, tanks
Averages	Missing score, weighted

How IES Questions Look

Word problems: real situations (shopping, travel, building) w/ 2–4 steps & traps at each step.

Core Approach

1. Read the last line 1st: know exactly what quantity & units the question wants you to find.

2. Write units on every number so they cancel & show which operation you need next.

3. Check: a sale price above the original price or a 40-hour walk means something went wrong.

⚠ Many wrong answers come from stopping 1 step early; reread the question.

✓ Slow, labeled work beats racing through the steps on IES questions every time.

Percentages

Core Formulas

Part = percent × whole: 15% of 80 = 0.15 × 80 = 12. In word problems, "of" means multiply.

Percent change = (new − old) ÷ old × 100. From 40 to 50: 10 ÷ 40 = a 25% increase.

Situation	Multiply by
Increase 20%	1.20
Decrease 20%	0.80
Tax 7%	1.07
30% off, then 10% off	0.70 × 0.90 = 0.63

Successive Changes

Multiply the factors: +10% then −10% → 1.10 × 0.90 = 0.99, which is a 1% drop, not 0% change.

Ex: a $120 jacket 30% off, then 10% off → 120 × 0.70 × 0.90 = $75.60, not 40% off ($72).

Compound growth: $P(1 + r)^t$. $500 at 4% a year for 3 years → $500(1.04)^3 \approx \$562.43$.

Reverse Percent

Find the original: after 25% off the price is $60 → original × 0.75 = 60 → original = $80.

Ex: $54 includes 8% tax → pre-tax price = 54 ÷ 1.08 = $50; don't just take 8% off $54.

Percent of a percent: 40% play sports & 25% of those play soccer → 0.40 × 0.25 = 10% of all.

⚠ Percent change divides by the ORIGINAL value, not the new one; that's the #1 trap.

✓ Use multipliers (1.15, 0.85) instead of finding the change & then adding it on.

Rates & Unit Conversions

Rates

Rate = amount ÷ time (or per unit): miles per hour, dollars per pound, pages per minute.

Unit rate compares deals: 12 oz for $3.00 = $0.25/oz; 16 oz for $3.60 = $0.225/oz.

Distance, Rate, Time

d = rt: 150 miles at 60 mph → t = 150 ÷ 60 = 2.5 hours. Change minutes to hours first.

Average speed = total distance ÷ total time, NOT the average of the 2 speeds in the problem.

Ex: 60 mi at 30 mph (2 h) + 60 mi at 60 mph (1 h) → 120 mi ÷ 3 h = 40 mph, not 45 mph.

Combined work: add rates; 1/4 + 1/6 = 5/12 per hour → together the job takes 2.4 hours.

Unit Conversion

Convert	Factor
ft → in	× 12
yd → ft	× 3
mi → ft	× 5,280
hr → min	× 60
ft^2 → in^2	× 144
yd^3 → ft^3	× 27

Chain factors so units cancel: 30 mi/h × 5,280 ft/mi ÷ 3,600 s/h = 44 ft/s. Write units each step.

⚠ Square & cubic units use the factor squared or cubed: 1 yd^2 = 9 ft^2 & 1 yd^3 = 27 ft^3.

✓ If the units don't cancel to what's asked, the setup is wrong; fix it before you calculate.

Ratios & Proportions

Ratios

Ratio 3 : 5 in a class of 32 → 8 parts of 4 each → 12 boys & 20 girls. Check: 12 + 20 = 32.

Part-to-whole: if boys : girls = 3 : 5, the boys' share of the whole class is 3/8, not 3/5.

3-part ratio: 2 : 3 : 5 of $500 → 10 parts, 1 part = $50 → shares of $100, $150 & $250.

Proportions

Match units: 3 cups / 4 servings = x cups / 10 servings → cross-multiply → x = 7.5 cups.

Scale Drawings & Maps

Scale 1 in : 20 ft → 3.5 in on the plan = 70 ft real. Area scales by k^2 & volume by k^3.

Map ex: scale 1 : 50,000 → 4 cm on the map = 200,000 cm = 2 km on the ground.

Direct vs. Inverse Variation

Type	Equation	Ex
Direct	y = kx	Pay vs. hours
Inverse	y = k/x	Workers vs. time

Inverse ex: 4 workers take 9 days → 6 workers take (4 × 9) ÷ 6 = 6 days (product stays the same).

Direct ex: $84 for 6 hours → $14/h → 15 hours earns $210 (y = 14x is direct variation).

Mixture: 2 L of 10% + 3 L of 20% → (0.2 + 0.6) ÷ 5 = 16%; add the amounts of the pure part.

⚠ Find the value of 1 part (total ÷ sum of the parts) before you do anything else.

✓ Label every fraction in a proportion w/ its units so the units line up across the = sign.

Area & Perimeter

Formulas to Memorize

Shape	Area	Perimeter
Rectangle	lw	2l + 2w
Triangle	½bh	Sum of sides
Trapezoid	$\frac{1}{2}(b_1 + b_2)h$	Sum of sides
Parallelogram	bh	2a + 2b
Circle	πr^2	2πr (circumference)

Composite Shapes

Add or subtract pieces: 10 × 6 rectangle + a semicircle w/ diameter 6 → 60 + ½π(3)² ≈ 74.1.

Shaded region = outer area − inner area: square of side 10 − circle of radius 5 → 100 − 25π.

Circle parts: sector area = (θ/360)πr²; arc length = (θ/360)2πr; 90° = ¼ of the circle.

Missing height? Use $a^2 + b^2 = c^2$; watch for 3-4-5 & 5-12-13 right triangles in figures.

Real-World Area

Ex: a 12 ft × 15 ft floor w/ tile at $3.50/$ft^2$ → 180 ft^2 × $3.50 = $630; check if it asks area or cost.

Round up for supplies: 1,200 ft^2 ÷ 350 ft^2 per gallon ≈ 3.4 → you must buy 4 gallons.

Scaling

Double every length → area × 4; triple → area × 9. Perimeter, a length, scales by k only.

⚠ Use the radius, not the diameter, in πr^2.

✓ Sketch & label the figure, then split odd shapes into rectangles & triangles to add up.

Surface Area & Volume

Volume Formulas

Solid	Volume
Rectangular prism	lwh
Cube	s^3
Cylinder	$\pi r^2 h$
Cone	$\frac{1}{3}\pi r^2 h$
Sphere	$(4/3)\pi r^3$
Pyramid	⅓Bh (B = base area)

Prism or cylinder: volume = base area × height. Cones & pyramids = ⅓ of the matching prism.

Surface Area

Box: 2lw + 2lh + 2wh. **Cylinder**: $2\pi r^2$ (top & bottom) + 2πrh (the curved side).

Applied Volume

Ex: a 4 × 3 × 2 ft tank = 24 ft^3; at 7.5 gal/ft^3 → 180 gallons; at 12 gal/min it fills in 15 min.

Fit cubes: 2-in cubes in a 6 × 8 × 10 in box → (3)(4)(5) = 60 cubes; divide each side.

Ex: cylinder r = 3 cm, h = 10 cm → 90π ≈ 282.7 cm^3 ≈ 283 mL, since 1 cm^3 = 1 mL.

Scaling Solids

Scale lengths by k → surface area × k^2 & volume × k^3; doubling a cube's side → volume × 8.

⚠ Volume is in cubic units & area is in square units; check the units on your answer.

✓ Memorize prism, cylinder & cone volume; there's no formula sheet on the ACT.

Averages & Number Forms

Mean, Median, Mode

Mean = sum ÷ count. **Median** = the middle value in order (average the 2 middle ones if even).

Missing value: scores 82, 90, 75; what 4th score gives a mean of 85? Sum must be 340 → 93.

Median ex: 3, 7, 8, 12, 20, 40 → middle values 8 & 12 → median 10; the outlier barely matters.

Weighted average: 20 students avg 80 & 30 avg 90 → (1,600 + 2,700) ÷ 50 = 86, not 85.

Change to data	Mean	Median
Add a high outlier	↑ a lot	↑ a little or not
Add 5 to every value	+5	+5
Double every value	× 2	× 2

Number Forms

Convert fractions ↔ decimals ↔ percents: 3/8 = 0.375 = 37.5%; use your calculator.

Scientific notation: 4.2×10^5 = 420,000; multiply → add exponents, divide → subtract.

Compare forms: 3/5, 0.58 or 61%? As decimals: 0.60, 0.58, 0.61 → 61% is the largest.

Rounding

Round only at the end; rounding mid-problem can land you on a nearby wrong answer.

Range = max − min; **mode** = most frequent.

⚠ A weighted average isn't the average of the averages unless groups are equal.

✓ Think in SUMS: total = mean × count.

Multistep Problems & Modeling

A 4-Step Method

1. Identify the goal. **2. List** the givens. **3. Plan** the steps. **4. Compute & check** the answer.

Ex: 28 mpg, $3.50/gal, 420-mile trip → 420 ÷ 28 = 15 gal × $3.50 = $52.50 for gas.

Modeling

Modeling (≥ 20% of questions) = turning a situation into an equation, graph or table.

Clue in words	Model
Starts at, plus per	Linear: y = mx + b
Grows by % each	Exponential: $a(1 + r)^x$
Halves, decays	$a(1 - r)^x$
Constant product	Inverse: y = k/x

Interpret: in C = 25 + 0.40m, 25 = the fixed fee & 0.40 = the cost per mile driven.

Ex exponential: 8,000 people growing 3% a year → $8{,}000(1.03)^t$; after 5 years ≈ 9,274.

Strategies

Backsolve: plug the answer choices into the problem; start w/ a middle value to save time.

Pick numbers: for percent problems w/ no values, start w/ a price of $100 & test the choices.

Pacing: ≈ 1 min per question; skip & return.

⚠ No penalty for guessing on the ACT: answer every question before time runs out.

✓ Use the calculator for arithmetic, but set up the units & steps on paper first to stay on track.