# The Ultimate ACT® Enhanced Math Integrating Essential Skills Cheat Sheet



## Integrating Essential Skills: Overview

### What It Tests

**IES**: multistep problems that combine basic skills: rates, percents, proportions, area, volume, averages.

**Weight**: ≈ 20% of the enhanced ACT math section, about 8 of 45 questions; modeling appears throughout.

**Format**: 45 questions in 50 minutes, 4 answer choices, calculator allowed, no formula sheet provided.

**Field test**: 4 of the 45 questions are unscored field-test items, so 41 count toward your score.

| Skill | Typical task |
|---|---|
| Percents | Discounts, tax, % change |
| Rates | Unit price, speed, work |
| Proportions | Scale, recipes, maps |
| Area/volume | Composite shapes, tanks |
| Averages | Missing score, weighted |

### How IES Questions Look

**Word problems**: real contexts (shopping, travel, construction) w/ 2–4 steps & distractors at each step.

**Chained steps**: the answer to step 1 (a rate) feeds step 2 (a total), so 1 slip spreads to the end; recheck each link.

### Core Approach

**1. Read the last line 1st**: know exactly what quantity & units the question wants (total? per unit?) before you start.

**2. Write units on every number** so they cancel correctly & show which operation you need next.

**3. Check reasonableness**: a discount price above the original or a 40-hour walk signals an error.

⚠ Wrong answers are often the result of stopping 1 step early; reread what the question asks for.

✓ IES questions reward care over cleverness; slow, labeled work beats racing through the steps.

## Percentages

### Core Formulas

**Part = percent × whole** (as a decimal): 15% of 80 = 0.15 × 80 = 12. "Of" means multiply; "is" means equals.

**Percent change** = (new − old) ÷ old × 100. 40 → 50 is a 25% increase, but 50 → 40 is a 20% decrease.

| Situation | Multiply by |
|---|---|
| Increase 20% | 1.20 |
| Decrease 20% | 0.80 |
| Tax 7% | 1.07 |
| 30% off, then 10% off | 0.70 × 0.90 = 0.63 |

### Successive Changes

**Multiply the factors**: +10% then −10% → 1.10 × 0.90 = 0.99, a 1% net DECREASE, not 0%; order doesn't matter.

**Compound growth**: $P(1 + r)^t$. $500 at 4% yearly for 3 years → $500(1.04)^3 \approx \$562.43$, not 500 + 3(20) = $560.

**Ex**: a $120 jacket is 30% off, then 10% off the sale price → 120 × 0.70 × 0.90 = $75.60, not 40% off ($72).

### Reverse Percent

**Find the original**: after a 25% discount the price is $60 → original × 0.75 = 60 → original = $80.

**Ex**: a price w/ 8% tax is $54 → pre-tax price = 54 ÷ 1.08 = $50; subtracting 8% of $54 gives $49.68, a trap.

### Percent of a Percent

**Ex**: 40% of students play sports & 25% of those play soccer → 0.40 × 0.25 = 0.10, so 10% of the whole school.

**Percentage points**: a rate rising from 20% to 25% is +5 points but a 25% increase (5 ÷ 20); know which is asked.

⚠ Percent change divides by the ORIGINAL value; dividing by the new value gives a common wrong answer.

✓ Use multipliers (1.15, 0.85) instead of computing the change & adding it; it's faster & safer.

## Rates & Unit Conversions

### Rates

**Rate** = amount ÷ time (or per unit): miles per hour, dollars per pound, pages per minute, liters per day.

**Unit rate** compares deals: 12 oz for $3.00 = $0.25/oz vs. 16 oz for $3.60 = $0.225/oz → the larger is cheaper.

### Distance, Rate, Time

**d = rt**: 150 miles at 60 mph → t = 150 ÷ 60 = 2.5 hours. Convert minutes to hours before plugging in.

**Average speed** = total distance ÷ total time, NOT the average of the 2 speeds. 60 mi at 30 mph + 60 at 60 → 40 mph.

**Ex**: 45 minutes at 48 mph → 0.75 h × 48 = 36 miles; using 45 instead of 0.75 gives 2,160, a giveaway error.

### Work Rates

**Combined work**: add rates. A fills a pool in 4 h (1/4 per h), B in 6 h (1/6) → together 5/12 per h → 2.4 h.

**Ex**: a printer makes 30 pages/min; 1,000 pages → 1,000 ÷ 30 ≈ 33.3 min. Rate × time = amount, always.

### Unit Conversion (Dimensional Analysis)

| Convert | Factor |
|---|---|
| ft → in | × 12 |
| yd → ft | × 3 |
| mi → ft | × 5,280 |
| hr → min | × 60 |
| $ft^2$ → $in^2$ | × 144 |
| $yd^3$ → $ft^3$ | × 27 |

**Chain factors** so units cancel: 30 mi/h × 5,280 ft/mi ÷ 3,600 s/h = 44 ft/s. Write units at every step.

**Metric**: 1 km = 1,000 m; 1 m = 100 cm; 1 L = 1,000 mL; 1 kg = 1,000 g; move the decimal by powers of 10.

⚠ Square & cubic units convert by the factor squared or cubed: 1 $yd^2$ = 9 $ft^2$ (not 3) & 1 $yd^3$ = 27 $ft^3$.

✓ If units don't cancel to what the question asks, the setup is wrong; fix it before you calculate anything.

## Ratios & Proportions

### Ratios

**Ratio** a:b compares parts. Boys : girls = 3 : 5 in a class of 32 → 8 parts of 4 → 12 boys & 20 girls (12 + 20 = 32).

**Part-to-whole**: if boys : girls = 3 : 5, the boys' share of the whole class is 3/8 (3 + 5 = 8 parts), not 3/5.

**3-part ratios**: a : b : c = 2 : 3 : 5 & a total of $500 → 10 parts, so 1 part = $50 → $100, $150 & $250.

### Proportions

**Set up a proportion** w/ matching units: 3 cups / 4 servings = x cups / 10 servings → x = 7.5 cups of flour.

**Cross-multiply** to solve: a/b = c/d → ad = bc. Keep the same order (top = cups, bottom = servings).

### Scale Drawings & Maps

**Scale** 1 in : 20 ft → 3.5 in on the plan = 70 ft in real life. Lengths scale by k, area by $k^2$ & volume by $k^3$.

**Ex**: a map scale of 1 : 50,000 → 4 cm on the map = 200,000 cm = 2 km on the ground; convert units last.

### Direct vs. Inverse Variation

| Type | Equation | Ex |
|---|---|---|
| Direct | y = kx | Pay vs. hours |
| Inverse | y = k/x | Workers vs. time |

**Inverse ex**: 4 workers take 9 days → 6 workers take (4 × 9) ÷ 6 = 6 days; total work (36 worker-days) stays fixed.

**Ex direct**: earning $84 for 6 hours → $14/h → 15 hours earns 15 × $14 = $210 (y = 14x, a direct variation).

### Mixtures

**Mixture**: amount × concentration adds: 2 L of 10% + 3 L of 20% → (0.2 + 0.6) ÷ 5 L = 16% solution.

⚠ Ratios give parts, not totals; find the value of 1 part (total ÷ sum of parts) before anything else.

✓ Label each fraction in a proportion w/ its units; mismatched units are the #1 proportion error.

## Area & Perimeter

### Formulas to Memorize (No Sheet)

| Shape | Area | Perimeter |
|---|---|---|
| Rectangle | lw | 2l + 2w |
| Triangle | ½bh | Sum of sides |
| Trapezoid | $\frac{1}{2}(b_1 + b_2)h$ | Sum of sides |
| Parallelogram | bh | 2a + 2b |
| Circle | $\pi r^2$ | $2\pi r$ (circumference) |

### Composite Shapes

**Add or subtract** pieces: a 10 × 6 rectangle w/ a semicircle of diameter 6 on top → 60 + ½π(3)² ≈ 74.1.

**Shaded region** = outer area − inner area. Square of side 10 minus a circle of radius 5 → 100 − 25π ≈ 21.5.

**Pythagorean help**: a missing height in a triangle or trapezoid often comes from $a^2 + b^2 = c^2$ (3-4-5, 5-12-13).

**Circle parts**: sector area = $(\theta/360)\pi r^2$; arc length = $(\theta/360)2\pi r$. A 90° sector is ¼ of the circle.

### Real-World Area

**Ex**: a 12 ft × 15 ft floor needs tile at $3.50/$ft^2$ → 180 $ft^2$ × $3.50 = $630. Read whether it asks cost or area.

**Ex**: paint covers 350 $ft^2$ per gallon; 1,200 $ft^2$ of wall → 1,200 ÷ 350 ≈ 3.4 → buy 4 gallons (round UP).

**Ex perimeter**: fencing a 30 ft × 20 ft yard on 3 sides (house is the 4th, a 30 ft side) → 30 + 20 + 20 = 70 ft.

### Scaling

**Double every length** → area × 4 ($k^2 = 2^2$); triple → area × 9. Perimeter is a length, so it scales by k only.

**Ex**: a photo enlarged from 4 × 6 to 8 × 12 has 4 times the area (24 → 96 $in^2$), not 2 times the original area.

⚠ Use radius, not diameter, in $\pi r^2$; questions often give the diameter to set this exact trap, so halve it 1st.

✓ Sketch & label the figure, even if one is drawn; split odd shapes into rectangles & triangles to find areas.

## Surface Area & Volume

### Volume Formulas

| Solid | Volume |
|---|---|
| Rectangular prism | lwh |
| Cube | $s^3$ |
| Cylinder | $\pi r^2 h$ |
| Cone | $\frac{1}{3}\pi r^2 h$ |
| Sphere | $(4/3)\pi r^3$ |
| Pyramid | ⅓Bh (B = base area) |

**Prism/cylinder rule**: volume = base area × height; cones & pyramids are ⅓ of the matching prism.

### Surface Area

**Rectangular box**: 2lw + 2lh + 2wh. **Cylinder**: $2\pi r^2$ (top & bottom) + $2\pi rh$ (the curved side, or lateral area).

**Open box** (no lid): subtract 1 lw face; read whether the problem includes the top before you add up the faces.

### Applied Volume

**Ex**: a tank 4 ft × 3 ft × 2 ft holds 24 $ft^3$; at 7.5 gal/$ft^3$ → 180 gallons. Convert after finding volume.

**Fill time**: 180 gal ÷ 12 gal/min = 15 min. Chain the volume answer straight into the rate step, w/ units.

**Ex**: cylinder r = 3 cm, h = 10 cm → π(9)(10) = 90π ≈ 282.7 $cm^3$; since 1 $cm^3$ = 1 mL, it holds ≈ 283 mL.

**Ex**: how many 2-in cubes fit in a 6 × 8 × 10 in box? (3)(4)(5) = 60 cubes; divide each dimension, not volumes.

### Scaling Solids

**Scale lengths by k** → surface area × $k^2$, volume × $k^3$. Doubling a cube's side → volume × 8.

⚠ Volume uses cubic units & area uses square units; a wrong exponent on units signals a wrong formula.

✓ Memorize prism, cylinder & cone volume; the ACT gives no formula sheet but often states sphere/cone formulas.

## Averages & Number Forms

### Mean, Median, Mode

**Mean** = sum ÷ count. **Median** = middle value in order (average the 2 middle ones if count is even).

**Missing value**: scores 82, 90, 75; what 4th score gives a mean of 85? Need sum 340 → 340 − 247 = 93.

**Median ex**: 3, 7, 8, 12, 20, 40 → middle values 8 & 12 → median = 10; the outlier 40 barely matters.

**Weighted average**: 20 students avg 80 & 30 students avg 90 → (20·80 + 30·90) ÷ 50 = 86, not 85.

**Change the mean**: 5 scores avg 80 (sum 400); a 6th score of 98 → 498 ÷ 6 = 83 → the mean rises by 3.

| Change to data | Mean | Median |
|---|---|---|
| Add a high outlier | ↑ a lot | ↑ a little or not |
| Add 5 to every value | +5 | +5 |
| Double every value | × 2 | × 2 |

### Number Forms

**Convert** fractions ↔ decimals ↔ percents: 3/8 = 0.375 = 37.5%. Calculators make this fast; use them.

**Scientific notation**: $4.2 \times 10^5 = 420{,}000$. Multiply: add exponents; divide: subtract exponents.

**Ex**: $0.000052 = 5.2 \times 10^{-5}$; move the decimal right → negative exponent; left → positive exponent.

**Compare forms**: which is largest: 3/5, 0.58 or 61%? Convert all to decimals: 0.60, 0.58, 0.61 → 61% wins.

**Ex**: $(6 \times 10^8) \div (3 \times 10^3) = 2 \times 10^5 = 200{,}000$.

### Estimation & Rounding

**Round only at the end**; rounding mid-problem can push you to a nearby wrong choice on multistep items.

⚠ Weighted averages aren't the average of the averages unless the groups are the same size.

✓ For mean problems, think in SUMS: total = mean × count; most missing-value problems fall fast this way.

## Multistep Problems & Modeling

### A 4-Step Method

**1. Identify** the goal & units. **2. List** the given values. **3. Plan** the chain of steps. **4. Compute & check** it.

**Ex**: a car gets 28 mpg; gas is $3.50/gal; the trip is 420 mi → 420 ÷ 28 = 15 gal × $3.50 = $52.50 for gas.

### Modeling

**Modeling** (≥ 20% of questions, across topics): turning a situation into an equation, graph or table.

| Clue in words | Model |
|---|---|
| Starts at, plus per | Linear: y = mx + b |
| Grows by % each | Exponential: $a(1 + r)^x$ |
| Halves, decays | $a(1 - r)^x$ |
| Constant product | Inverse: y = k/x |

**Interpret**: in C = 25 + 0.40m, 25 = fixed fee & 0.40 = cost per mile; questions ask what each part means.

**Ex exponential**: a town of 8,000 grows 3% per year → $P = 8{,}000(1.03)^t$; after 5 years ≈ 9,274 people.

**Graph models**: slope = rate of change; y-intercept = starting value; read units on both axes before choosing.

### Strategies

**Backsolve**: plug answer choices into the problem; start w/ a middle value to see if you need bigger or smaller.

**Pick numbers**: for percent problems w/ no values given, assume a starting price of $100 to test the choices.

**Pacing**: ≈ 1 minute per question; skip a long multistep problem & return after the quick ones.

**Calculator**: store intermediate results in memory instead of retyping rounded values; it avoids drift.

**Check the choices**: if 2 answers differ by a factor of 10, 12 or 60, recheck each of your unit conversions.

⚠ No penalty for guessing on the ACT: answer every question, even the ones you skip at first.

✓ Use your calculator for arithmetic, but set up the units & steps on paper first so you don't get lost.