The Simple ACT® Enhanced Math Number and Quantity Cheat Sheet

Number Sets & Properties

The Number Sets

Set	Examples	Notes
Integers	…−2, 0, 5	No fractions
Rational	3/4, −0.5	a/b, b ≠ 0
Irrational	π, √2	Never ends/repeats
Real	All above	On number line
Complex	3 + 2i	a + bi

Rational (fraction of integers): decimal ends (0.25) or repeats (0.333…). π never does.
√n is rational only if n is a perfect square (√49 = 7 is rational; √50 ≈ 7.07 is irrational).

Factors & Primes

Prime: exactly 2 factors (2, 3, 5, 7, 11…). 1 is NOT prime; 2 is the only even prime.
GCF (greatest common factor): shared primes, lowest powers. **LCM** (least common multiple): all primes, highest powers, used once each.
e.g. $72 = 2^3 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$ → GCF = $2^2 \cdot 3 = 12$; LCM = $2^3 \cdot 3^2 \cdot 5 = 360$. Check: 12 × 360 = 72 × 60.
# of factors: add 1 to each exponent, multiply. $72 = 2^3 \cdot 3^2$ → (3+1)(2+1) = 12 factors.

Even & Odd

Parity: odd + odd = even; odd + even = odd; odd × odd = odd; any × even = even.

⚠ 0 is an integer, even, and rational, but not positive or negative. 1 isn't prime.

✓ "Must be true" Qs: test 0, 1, −1, a fraction and a negative; 1 miss kills a choice.

Exponent Rules

Core Rules (same base!)

Rule	Form	Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^7$
Quotient	$a^m / a^n = a^{m-n}$	$x^8 / x^2 = x^6$
Power	$(a^m)^n = a^{mn}$	$(x^2)^5 = x^{10}$
Zero	$a^0 = 1$	$7^0 = 1$
Negative	$a^{-n} = 1/a^n$	$2^{-3} = 1/8$

Common Moves

Same base: $8^x = 4^{x+1} \rightarrow 2^{3x} = 2^{2x+2} \rightarrow 3x = 2x + 2 \rightarrow x = 2$. Rewrite both as powers of 2 first.
Coefficients multiply, exponents add: $(3x^2)(4x^5) = 12x^7$, not $7x^7$ and not $12x^{10}$. Keep them separate.
Negative base: $(-2)^4 = 16$ but $-2^4 = -16$ (the exponent hits only the 2, not the minus sign).
Factor out the smaller power: $3^{x+2} - 3^x = 3^x(3^2 - 1) = 3^x(9 - 1) = 8 \cdot 3^x$. Same move for 2^x, 5^x.
Base 0–1: powers shrink, $(0.5)^3 = 0.125$.

Traps

No sum rule: $(a+b)^2 \neq a^2 + b^2$. It's $a^2 + 2ab + b^2$.
Add same powers: $2^3 + 2^3 = 2 \cdot 2^3 = 2^4$, not 2^6.
Same exponent: $a^n b^n = (ab)^n$, e.g. $2^5 \cdot 5^5 = 10^5$.

⚠ A negative exponent means reciprocal, not a negative number: $2^{-3} = 1/8$, a positive number.

✓ Stuck? Plug x = 2 into the question and into each choice; the choice that matches wins.

Radicals & Rational Exponents

Radical ↔ Exponent

Rule: $a^{m/n} = (\sqrt[n]{a})^m$. Bottom = root, top = power.
e.g. $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$. Take the root first.

Radical	Exponent	Value
$\sqrt{x}$	$x^{1/2}$	$\sqrt{49} = 7$
$\sqrt[3]{x}$	$x^{1/3}$	$\sqrt[3]{-8} = -2$
$1/\sqrt{x}$	$x^{-1/2}$	$9^{-1/2} = 1/3$

Simplifying

Pull out squares: $\sqrt{72} = \sqrt{(36 \cdot 2)} = 6\sqrt{2}$.
Add like radicals only: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$ (1.41 + 1.73 = 3.14, not 2.24).
Multiply: $\sqrt{a} \cdot \sqrt{b} = \sqrt{(ab)}$, e.g. $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$.
Simplify first, then add: $\sqrt{12} + \sqrt{27} = \sqrt{(4 \cdot 3)} + \sqrt{(9 \cdot 3)} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. Pull out squares.

Rationalizing

Rationalize (no √ in the denominator): $6/\sqrt{3} \cdot \sqrt{3}/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$. Multiply top and bottom by √3.
Two-term bottom: multiply by the conjugate. $4/(3-\sqrt{5}) \cdot (3+\sqrt{5})/(3+\sqrt{5}) = 3 + \sqrt{5}$.
Radical eq.: isolate √, square both sides, then CHECK each answer in the original.
e.g. $\sqrt{(x+6)} = x \rightarrow x^2 - x - 6 = 0 \rightarrow x = 3$ or -2. Check: only x = 3 works ($\sqrt{4} \neq -2$).
Even roots of negatives aren't real: $\sqrt{-9} = 3i$, an imaginary number. $\sqrt[3]{-8} = -2$ is real.

⚠ $\sqrt{(a+b)} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{(9+16)} = 5$, not 7.

✓ Radical answer choices? Compare decimals on your calculator to find the match.

Sci. Notation, Abs. Value & Order

Scientific Notation

Form: $a \times 10^n$, $1 \leq |a| < 10$. $0.00045 = 4.5 \times 10^{-4}$.
Multiply: multiply a's, add exponents. $(3 \times 10^5)(4 \times 10^2) = 12 \times 10^7 = 1.2 \times 10^8$ (shift 1 place).
Divide: divide a's, subtract exponents. $(6 \times 10^8)/(2 \times 10^3) = 3 \times 10^5$. Top minus bottom.
Add: match the powers of 10 first. $5 \times 10^4 + 3 \times 10^3 = 5 \times 10^4 + 0.3 \times 10^4 = 5.3 \times 10^4$. Don't add exponents.

Absolute Value

$|x|$ = distance from 0, never negative.

Form	Solution
$\|x\| = 5$	x = 5 or −5
$\|x - 2\| < 3$	−1 < x < 5
$\|x - 2\| > 3$	x < −1 or x > 5
$\|x\| = -4$	No solution

Isolate |…| first: $2|x+1| - 3 = 7 \rightarrow |x+1| = 5 \rightarrow x + 1 = 5$ or $-5 \rightarrow x = 4$ or $x = -6$. Check both.
Two cases: $|2x - 1| = 7 \rightarrow 2x - 1 = 7$ or $2x - 1 = -7 \rightarrow x = 4$ or $x = -3$. Check both.

Ordering Numbers

Compare: turn every number into a decimal on the calculator, then put them in order.
Between 0 and 1: $x^2 < x < \sqrt{x}$ (squaring makes it smaller): $0.25 < 0.5 < 0.71$ for x = 0.5.
Negatives: −0.1 > −0.5 (farther left on the number line = smaller number). −5 < −1.
Midpoint of a and b = (a + b)/2: −3 and 8 → 2.5.

⚠ 12×10^7 isn't scientific notation: shift to 1.2×10^8.

✓ Use the calculator's EE key for powers of 10.

Complex Numbers

Basics

i = $\sqrt{-1}$, so $i^2 = -1$. **Complex #**: a + bi (a = real part, b = imaginary part). 3 − 2i: a = 3, b = −2.
√ of a negative: $\sqrt{-36} = \sqrt{36} \cdot i = 6i$.

Power	Value	Remainder
i^1	i	1
i^2	−1	2
i^3	−i	3
i^4	1	0

Big powers: divide by 4, use the remainder. i^{27}: 27 = 4·6 + 3 → $i^3 = -i$. Remainder 0 → $i^4 = 1$.
Two negative roots: $\sqrt{-4} \cdot \sqrt{-9} = (2i)(3i) = 6i^2 = -6$, NOT 6. Always pull out i before multiplying.

Operations

Add: combine like parts. (3+2i) + (1−5i) = 4 − 3i.
Multiply: FOIL, then $i^2 \rightarrow -1$. (2+3i)(1−i) = 5 + i.
Conjugate of a+bi is a−bi. Multiply top and bottom by it to divide; the bottom becomes real.
Divide: $(5+i)/(1+i) \cdot (1-i)/(1-i) = (6 - 4i)/2 = 3 - 2i$.
(a+bi)(a−bi) = $a^2 + b^2$, a real number.

Equal Complex Numbers

Match parts: a + 4i = 7 + bi → a = 7, b = 4.
Complex plane: plot a + bi at (a, b). Size: $|a + bi| = \sqrt{(a^2 + b^2)}$, so $|3 - 4i| = \sqrt{25} = 5$.
Quadratics: if $b^2 - 4ac < 0$, the roots are complex, a pair p ± qi. $x^2 + 4 = 0 \rightarrow x = 2i$ or $-2i$.

⚠ Never leave i^2 in an answer: $(4i)^2 = 16i^2 = -16$.

✓ "a + bi form" means no i in the denominator.

Vectors

Components

Vector: has size and direction. **v** = ⟨a, b⟩ = a**i** + b**j**.
From P to Q: end minus start. P(1, 2), Q(4, 6) → **PQ** = ⟨4 − 1, 6 − 2⟩ = ⟨3, 4⟩, length $\sqrt{25} = 5$.
Magnitude (length): $\|\mathbf{v}\| = \sqrt{(a^2 + b^2)}$. ⟨3, 4⟩ → 5.

Operations

Operation	Rule	u=⟨2,3⟩
u + **v**	Add parts	+⟨1,−4⟩=⟨3,−1⟩
k**u**	Scale parts	3**u** = ⟨6,9⟩
−**u**	Flip dir.	⟨−2,−3⟩

Subtract: ⟨2, 3⟩ − ⟨1, −4⟩ = ⟨2 − 1, 3 − (−4)⟩ = ⟨1, 7⟩ (subtract each part; watch double negatives).
Head-to-tail: start **v** at the tip of **u**; the sum runs from the start of **u** to the tip of **v**.
Unit vector (length 1): divide by the length. ⟨3, 4⟩/5 = ⟨0.6, 0.8⟩, same direction as ⟨3, 4⟩.

Applications

Resultant (net force, plane + wind): add the components, then find the length.
Scale: k < 0 flips the direction; $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$.
Direction: tan θ = b/a; then check which quadrant the vector really points to (signs of a, b).
Equal vectors: same parts (same length and direction), from any start point.
Parallel: **v** = k**u**. ⟨2, 3⟩ ∥ ⟨−4, −6⟩ (k = −2).

⚠ Lengths don't just add: $\|\mathbf{u} + \mathbf{v}\| \neq \|\mathbf{u}\| + \|\mathbf{v}\|$. Add the parts first, then find the length.

✓ Sketch the arrows to catch sign errors.

Matrices

Size & Basics

Dimensions: rows × columns. A 2×3 matrix has 2 rows and 3 columns (6 entries).
Add/subtract: same size only; combine matching entries. Different sizes: undefined.
Scalar: multiply every entry. 3[2 −1] = [6 −3].

Multiplying Matrices

Rule: (m×**n**)(**n**×p) → m×p. Inner numbers must match; outer numbers = the answer's size.
Entry: row of A · column of B (multiply pairs, add).

A	B	AB?
2×3	3×4	Yes → 2×4
3×2	3×2	No
1×3	3×1	Yes → 1×1

e.g. [1 2; 3 4]·[5; 6] = [1·5 + 2·6; 3·5 + 4·6] = [17; 39], a 2×1 matrix: (2×2)(2×1) → 2×1.
Order matters: AB usually ≠ BA.

Determinant (2×2)

det [a b; c d] = ad − bc. If det = 0, no inverse.
Inverse: $A^{-1} = (1/(ad - bc)) \cdot [d\ -b;\ -c\ a]$. Swap a and d; negate b and c; divide by det.
Identity I = [1 0; 0 1]: AI = A, and $AA^{-1} = I$.
Matrix equations: match entries in the same spots. [x 3] = [5 3] → x = 5 (3 = 3 already).

⚠ Matrix multiplication is row × column, summed. It is NOT multiplying matching entries.

✓ Use your calculator's matrix menu.

Units, Strategy & Traps

Units

Convert: multiply by fractions equal to 1 and cancel units: 2 yd · (3 ft/1 yd) = 6 ft.
Area units: 1 yd = 3 ft, so 1 $yd^2 = 9$ ft^2.
Round only at the end; match the units and decimal places in the answer choices.

Which Tool?

You see…	Do this
$x^{a/b}$	Root ↔ exponent
Bases 8, 4, 2	Rewrite as base 2
i^n, big n	n ÷ 4, remainder
⟨a, b⟩	Work by parts
Grid of #s	Check sizes 1st

Test-Day Moves

Pacing: 45 Qs in 50 min ≈ 67 s each. Qs get harder, so move fast on the early ones.
Backsolve: plug in the 4 choices; start with a middle one, then go up or down.
Guess: no penalty, so answer every Q.
Pick numbers for answers with variables (x = 2); test every choice, not just the first that works.
Calculator: allowed on the whole section. Learn its complex number and matrix modes.

⚠ No formula sheet on the ACT. Memorize these rules: exponents, i-cycle, det, magnitude.

✓ Reread the last line: what exactly is the Q asking for (x, x^2, or a+bi form)?