The Ultimate ACT® Enhanced Math Number and Quantity Cheat Sheet

Number Sets & Properties

The Number Sets

Set	Examples	Notes
Natural	1, 2, 3…	No 0, no neg.
Integers	…−2, 0, 5	No fractions
Rational	3/4, −0.5, 0.333…	a/b, b ≠ 0
Irrational	π, √2, e	Never ends/repeats
Real	All above	Rat. ∪ irrat.
Complex	3 + 2i	a + bi; reals: b=0

Rational: decimal ends (0.25) or repeats (0.1666…). **√n** is rational only if n is a perfect square (√50 irrat.).
Closure: rat. ± × ÷ rat. = rat. Rat. (≠0) × irrat. = irrat. Irrat. + irrat. can be rational: (2+√3)+(2−√3)=4.

Factors, Primes & Multiples

Prime: exactly 2 factors (2, 3, 5, 7, 11, 13…). 1 is NOT prime; 2 is the only even prime; 51 = 3·17 isn't.
GCF: shared primes, lowest powers. **LCM**: all primes, highest powers. GCF × LCM = a × b for 2 numbers.
e.g. 72 = $2^3 \cdot 3^2$, 60 = $2^2 \cdot 3 \cdot 5$ → GCF = $2^2 \cdot 3$ = 12; LCM = $2^3 \cdot 3^2 \cdot 5$ = 360. Check: 12·360 = 72·60 = 4,320 ✓.
# of factors: add 1 to each exponent, multiply. 72 = $2^3 \cdot 3^2$ → (3+1)(2+1) = 12 factors, from 1 up through 72.

Divisibility & Parity

By 3/9: digit sum ÷ by 3/9. **By 4**: last 2 digits ÷ by 4. **By 6**: by 2 AND 3. **By 5**: ends in 0 or 5. **By 10**: ends in 0.
Parity: even ± even = even; odd ± odd = even; odd × odd = odd; any × even = even; odd ± even = odd.
Remainders: n = dq + r, 0 ≤ r < d. Remainders repeat in cycles → find the cycle length, then use n ÷ length.

⚠ 0 is an integer, even, and rational, but not positive. "Consecutive integers" can include negatives.

✓ "Must be true" Qs: test 0, 1, −1, a fraction (½) and a negative before picking; 1 counterexample kills a choice.

Exponent Rules

Core Rules (same base!)

Rule	Form	Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^7$
Quotient	$a^m / a^n = a^{m-n}$	$x^8 / x^2 = x^6$
Power	$(a^m)^n = a^{mn}$	$(x^2)^5 = x^{10}$
Prod. power	$(ab)^n = a^n b^n$	$(2x)^3 = 8x^3$
Zero	$a^0 = 1\ (a \neq 0)$	$7^0 = 1$
Negative	$a^{-n} = 1/a^n$	$2^{-3} = 1/8$
Fraction	$(a/b)^{-n} = (b/a)^n$	$(2/3)^{-2} = 9/4$

Common Moves

Rewrite to same base: $8^x = 4^{x+1} \rightarrow 2^{3x} = 2^{2x+2}$ → set exponents equal: 3x = 2x + 2 → x = 2. Check: $8^2 = 4^3 = 64$.
Factor out the smaller power: $3^{x+2} - 3^x = 3^x(9 - 1) = 8 \cdot 3^x$.
Coefficients multiply, exponents add: $(3x^2y)(4x^5y^3) = 12x^7y^4$.
Neg. base: $(-2)^4 = 16$ but $-2^4 = -16$ (exponent hits the 2 only). Even power → positive; odd power keeps the sign.
Base 0–1: powers shrink: $(0.5)^3 = 0.125 < 0.5$. Base > 1 grows.

Traps

No sum rule: $(a+b)^2 \neq a^2 + b^2$; it's $a^2 + 2ab + b^2$. $2^3 + 2^3 = 2 \cdot 2^3 = 2^4$, not 2^6; you add terms, don't multiply.
Same exponent: $a^n b^n = (ab)^n$, e.g. $2^5 \cdot 5^5 = 10^5 = 100{,}000$.
Compare sizes: match exponents. 2^{30} vs $3^{20} \rightarrow (2^3)^{10}$ vs $(3^2)^{10} \rightarrow 8^{10}$ vs $9^{10} \rightarrow 3^{20}$ is bigger, b/c 9 > 8.
Special bases: $1^n = 1$; $0^n = 0$ (n > 0). $(-1)^n = \pm 1$ by parity.

⚠ a^{-n} is NOT negative: $2^{-3} = 1/8 > 0$. A negative exponent means reciprocal; the sign stays the same.

✓ Lost? Plug a small value (x = 2) into the stem and each choice; the choice that matches the stem's number wins.

Radicals & Rational Exponents

Radical ↔ Exponent

Rule: $a^{m/n} = \sqrt[n]{(a^m)} = (\sqrt[n]{a})^m$. Denominator = root, numerator = power. Works for any rational exponent.
e.g. $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$. $16^{-3/4} = 1/(\sqrt[4]{16})^3 = 1/8$.
Tip: take the root first; numbers stay small.

Radical	Exponent	Value
$\sqrt{x}$	$x^{1/2}$	$\sqrt{49} = 7$
$\sqrt[3]{x}$	$x^{1/3}$	$\sqrt[3]{(-8)} = -2$
$\sqrt{(x^3)}$	$x^{3/2}$	$4^{3/2} = 8$
$1/\sqrt{x}$	$x^{-1/2}$	$9^{-1/2} = 1/3$

Simplifying

Pull out squares: $\sqrt{72} = \sqrt{(36 \cdot 2)} = 6\sqrt{2}$. $\sqrt{(50x^3)} = 5x\sqrt{(2x)}$.
Multiply: $\sqrt{a} \cdot \sqrt{b} = \sqrt{(ab)}$. **Divide**: $\sqrt{a}/\sqrt{b} = \sqrt{(a/b)}$ (a,b ≥ 0).
Add like radicals only: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$.
Simplify first: $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.

Rationalizing

One term: $6/\sqrt{3} \cdot \sqrt{3}/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$.
Binomial: × conjugate. $4/(3-\sqrt{5}) \cdot (3+\sqrt{5})/(3+\sqrt{5}) = 4(3+\sqrt{5})/(9-5) = 3+\sqrt{5}$. Bottom becomes a rational number.
Conjugate product: $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$, b/c the middle terms cancel. $(5+\sqrt{3})(5-\sqrt{3}) = 25 - 3 = 22$.

Roots & Equations

Odd roots of negatives are real: $\sqrt[3]{(-27)} = -3$. **Even roots** of negatives → imaginary: $\sqrt{-9} = 3i$, not −3.
Radical eq.: isolate √, square both sides, CHECK. $\sqrt{(x+6)} = x \rightarrow x^2 - x - 6 = 0 \rightarrow x = 3$ (x = −2 fails).
Estimate: √50 is between 7 and 8, close to 7 (≈ 7.07).

⚠ $\sqrt{(x^2)} = |x|$, not x (x = −3 → √9 = 3). And $\sqrt{(a+b)} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{(9+16)} = \sqrt{25} = 5$, not 3 + 4 = 7.

✓ Answer choices in radical form? Check w/ the calculator: compare each choice's decimal to your answer's.

Sci. Notation, Abs. Value & Order

Scientific Notation

Form: a × 10^n, 1 ≤ |a| < 10, n an integer. 0.00045 = 4.5 × 10^{-4}.
Multiply: multiply a's, add exponents. $(3\times10^5)(4\times10^2) = 12\times10^7 = 1.2\times10^8$ (shift so 1 ≤ a < 10, exponent +1).
Divide: divide a's, subtract exponents. $(6\times10^8)/(2\times10^3) = 3\times10^5$; $(2\times10^3)/(8\times10^5) = 0.25\times10^{-2} = 2.5\times10^{-3}$.
Add: match powers first. $5\times10^4 + 3\times10^3 = 5\times10^4 + 0.3\times10^4 = 5.3\times10^4$. Never add the exponents when adding.
Compare: normalize, then the bigger exponent wins: $9.9\times10^3 < 1.1\times10^4$. A negative exponent → value < 1.

Absolute Value

|x| = distance from 0, always ≥ 0. |a − b| = distance between a and b on the number line: |−3 − 5| = 8.

Form	Means	Solution
$\|x\| = 5$	Two points	x = ±5
$\|x - 2\| < 3$	Within 3 of 2	−1 < x < 5
$\|x - 2\| > 3$	> 3 from 2	x < −1 or x > 5
$\|x\| = -4$	Impossible	No solution

Isolate |…| first, then split into 2 cases: $2|x+1| - 3 = 7 \rightarrow |x+1| = 5 \rightarrow x + 1 = \pm5 \rightarrow x = 4$ or x = −6. Check both.
Two-case eq.: $|2x - 1| = x + 4 \rightarrow 2x - 1 = x + 4$ or $-(x + 4) \rightarrow x = 5$ or −1. Plug back in: RHS must be ≥ 0.

Ordering Numbers

Compare: convert all to decimals (calc). 3/7 ≈ 0.43, √0.2 ≈ 0.45, 0.444… ≈ 0.44 → 3/7 < 0.444… < √0.2.
Between 0 and 1: $x^3 < x^2 < x < \sqrt{x} < 1/x$ (x = ¼: 1/64 < 1/16 < ¼ < ½ < 4). The order reverses for any x > 1.
Midpoint of a and b on a number line = (a+b)/2: −3 and 8 → 2.5. Distance between them = |8 − (−3)| = 11.

⚠ 12×10^7 isn't sci. notation: shift to 1.2×10^8. Check 1 ≤ |a| < 10 before matching your answer to the choices.

✓ Use the calc's EE key for powers of 10 (3EE5 = 3×10^5); it avoids missing-parenthesis errors on division.

Complex Numbers

Basics

i = √−1, so $i^2 = -1$. **Complex #**: a + bi (a = real part, b = imaginary part). Every real # is complex (b = 0).
√ of negatives: pull out i first. √−36 = 6i. √−4·√−9 = (2i)(3i) = $6i^2$ = −6, NOT √36 = 6. The product rule fails here.

Power	Value	Shortcut
i^1	i	rem. 1
i^2	−1	rem. 2
i^3	−i	rem. 3
i^4	1	rem. 0

Big powers: divide the exponent by 4, use the remainder. i^{27}: 27 = 4·6 + 3 → i^3 = −i. i^{100} = 1 (remainder 0).

Operations

Add/subtract: combine like parts. (3+2i) − (1−5i) = 2 + 7i.
Multiply: FOIL, then $i^2 \rightarrow -1$. (2+3i)(1−i) = 2 − 2i + 3i − $3i^2$ = 2 + i + 3 = 5 + i. Combine real, then imag.
Conjugate of a+bi is a−bi. (a+bi)(a−bi) = $a^2 + b^2$ (real!).
Divide: × conjugate of the denominator. (5+i)/(1+i) · (1−i)/(1−i) = (6−4i)/2 = 3 − 2i. Bottom: $1^2 + 1^2 = 2$.

Complex Plane & Equations

Plot a+bi at (a, b): x-axis real, y-axis imaginary. **|a+bi|** = $\sqrt{(a^2+b^2)}$ = distance from 0: |3−4i| = 5.
Equal complex #s: real = real AND imag. = imag. a + 4i = 7 + bi → a = 7, b = 4. Set up 2 equations, solve each.
Quadratic roots: $b^2 - 4ac < 0 \rightarrow$ 2 complex roots, a conjugate pair p ± qi. $x^2 + 4 = 0 \rightarrow x = \pm2i$.
Conjugate roots: sum (p+qi)+(p−qi) = 2p; product = $p^2 + q^2$.
Cycle sum: i + i^2 + i^3 + i^4 = i − 1 − i + 1 = 0, so any 4 consecutive powers of i sum to 0. Group a long sum in 4s.
Pure imaginary: a = 0, b ≠ 0 (e.g. 5i). **Real**: b = 0.

⚠ Never leave i^2 in an answer. It's −1, so $(4i)^2 = 16i^2 = -16$.

✓ "Write in a+bi form" = rationalize: no i in the denominator.

Vectors

Components & Notation

Vector: magnitude + direction. **v** = ⟨a, b⟩ = a**i** + b**j** (**i**, **j** = unit vectors along the x- and y-axes). ⟨3, −2⟩ = 3**i** − 2**j**.
From P(x_1, y_1) to Q(x_2, y_2): **PQ** = ⟨$x_2 - x_1$, $y_2 - y_1$⟩. End minus start. P(1, 2), Q(4, 6) → ⟨3, 4⟩, length 5.
Magnitude: $\|\mathbf{v}\| = \sqrt{(a^2 + b^2)}$. ⟨−5, 12⟩ → √(25+144) = 13.
Direction: tan θ = b/a; check the quadrant. Components: a = $\|\mathbf{v}\|\cos\theta$, b = $\|\mathbf{v}\|\sin\theta$ (θ from +x-axis).

Operations

Operation	Rule	e.g. u=⟨2,3⟩
u + **v**	Add parts	+⟨1,−4⟩ = ⟨3,−1⟩
u − **v**	Subtract parts	−⟨1,−4⟩ = ⟨1,7⟩
k**u**	Scale each part	3**u** = ⟨6,9⟩
−**u**	Opposite dir.	⟨−2,−3⟩
Unit **u**	**u** / $\|\mathbf{u}\|$	⟨2,3⟩/√13

Head-to-tail: draw **v** starting at the tip of **u**; **u**+**v** runs from the tail of **u** → the tip of **v**. Order doesn't matter.
Parallelogram: **u**+**v** and **u**−**v** are its diagonals.

Applications

Resultant (net force, velocity w/ wind): add components, then find the magnitude and direction.
Scale: k > 1 longer, 0 < k < 1 shorter, k < 0 flips direction. $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$: $\|-2\mathbf{v}\| = 2\|\mathbf{v}\|$, pointing the other way.
Equal vectors: same components (same length + direction), any start point. Position doesn't matter.
Zero vector ⟨0, 0⟩: **u** + (−**u**) = **0**. Vectors forming a closed loop (triangle, polygon) always sum to **0** = ⟨0, 0⟩.
Parallel: **v** = k**u** for some k. ⟨2, 3⟩ ∥ ⟨−4, −6⟩ (k = −2).

⚠ $\|\mathbf{u}+\mathbf{v}\| \neq \|\mathbf{u}\| + \|\mathbf{v}\|$ unless they point the same way. Add the parts first, then take the length of the sum.

✓ Sketch it. A quick arrow diagram catches sign and quadrant errors before you pick a choice.

Matrices

Size & Basic Operations

Dimensions: rows × columns (m × n). Entry a_{ij} = row i, column j. A 2×3 has 2 rows, 3 columns.
Add/subtract: same dimensions only; combine matching entries. A 2×2 + a 2×3 is undefined: no answer exists.
Scalar: k × each entry. 3[2 −1] = [6 −3].

Matrix Multiplication

Rule: (m×**n**)(**n**×p) → m×p. Inner #s must match; outer #s = the answer's size. (2×3)(3×4) → 2×4.
Entry (i, j) = row i of A · column j of B: multiply pairs, add.
e.g. [1 2; 3 4]·[5; 6] = [1·5+2·6; 3·5+4·6] = [17; 39] (2×1).

A	B	AB?
2×3	3×4	Yes → 2×4
3×2	3×2	No (2≠3)
1×3	3×1	Yes → 1×1
3×1	1×3	Yes → 3×3

Order matters: AB ≠ BA in general (one may not even exist).

Determinant & Inverse (2×2)

det [a b; c d] = ad − bc. det = 0 → no inverse (singular).
Inverse: A^{-1} = (1/(ad−bc))·[d −b; −c a]. Swap a,d; negate b,c.
Identity I = [1 0; 0 1]: AI = IA = A; AA^{-1} = I.
Systems: AX = B → X = A^{-1}B. Use calc matrix menu.
Matrix equations: [x 3; 2 y] = [5 3; 2 −1] → match entries in the same spots → x = 5, y = −1. Equal = every entry equal.
Squaring: A^2 = A·A exists only if A is square (n × n).

⚠ Multiplying ≠ multiplying matching entries. It's each row × each column, summed, for every entry.

✓ Asked for only 1 entry? Compute just that row · column; skip the rest. Saves a full minute on test day.

Units, Strategy & Traps

Units & Quantities

Convert w/ fractions = 1; cancel units. 60 mi/hr · (5280 ft/1 mi) · (1 hr/3600 s) = 88 ft/s. Units cancel diagonally.
Squared/cubed units: 1 yd = 3 ft → 1 $yd^2 = 3^2 = 9$ ft^2, 1 $yd^3 = 3^3 = 27$ ft^3. Square or cube the factor too.
Precision: round only at the end; match the answer's units and decimal places. Early rounding shifts answers.

Which Tool?

You see…	Do this
$x^{a/b}$ or $\sqrt[n]{}$	Root ↔ exponent
Diff. bases (8, 4, 2)	Rewrite as one base
i^n, n large	n ÷ 4, use remainder
i in denominator	× conjugate
⟨a, b⟩ or arrows	Work by components
[ ] grid of #s	Check dimensions 1st
Huge/tiny #s	Sci. notation + EE key

Test-Day Moves

Pacing: 45 Qs, 50 min ≈ 67 s each. Qs get harder → bank time on the early Qs for the last 10–15 hard ones.
Backsolve: plug the 4 choices in; start w/ a middle value.
Pick numbers for variable answers; test every choice.
Calculator: allowed all section; complex, matrix & radical modes save time. Know them before test day.
No guessing penalty: answer every Q; 4 choices = 25% blind odds, better after cutting 1–2 choices.
Mark & return: flag any Q past ~2 min; come back after the last Q. Never let 1 Q eat 5 minutes of the 50.

⚠ No formula sheet on the ACT. Memorize exponent rules, the i-cycle, magnitude, det and unit factors.

✓ Reread the last line of the Q: "value of x^2" or "in a+bi form" decides which choice is right. Answer what's asked.