The Simple ACT® Enhanced Math Statistics and Probability Cheat Sheet

Center & Spread

Measures of Center

Mean = sum ÷ count: 4, 6, 8, 10, 22 → 50/5 = 10.
Median = middle value once sorted → 8; even count → average the 2 middle values.
Mode = the most frequent value.
Even count: 3, 5, 9, 11 → median = (5 + 9)/2 = 7, even though 7 isn't in the data.
Missing value: mean of 5 #s = 12 → sum = 60; 4 known #s sum to 47 → 5th # = 13.
Which center? Symmetric → mean; skewed or outliers → median (it resists outliers).
Weighted mean: 20 students avg 80 + 30 avg 90 → (1,600 + 2,700)/50 = 86, not 85.

Measures of Spread

Range = max − min. **IQR** = Q3 − Q1, the middle 50% of the data (resists outliers).
Standard deviation (SD): typical distance from the mean; bigger SD → more spread.
SD = 0 only when every value is the same (5, 5, 5); SD is never negative; adding k keeps SD.

Changing the Data

Change	Center	Spread
Add k to all	+ k	Same
Multiply by k	× k	× k
Add outlier	Mean moves	Range ↑

⚠ An outlier pulls the mean, not the median.

✓ Use sums for missing-value Qs: mean × count = sum, then subtract the known values.

Distributions & Displays

Reading Displays

Display	Shows best
Dot plot	Each value
Histogram	Shape, counts
Box plot	5-number summary
Bar chart	Categories

5-number summary: min, Q1, median, Q3, max; the box = IQR, its middle line = median.
Outlier rule: below Q1 − 1.5·IQR or above Q3 + 1.5·IQR; drawn as a dot past the whisker.
Histogram: bar heights = counts per interval; look for the peak, the spread & the tail.

Shape

Symmetric: mean ≈ median. **Right-skewed** (tail right): mean > median (pulled right).
Left-skewed (tail left): mean < median; the mean always follows the tail of the data.

Normal Distribution

Bell curve: symmetric; mean = median = mode, all at the center peak of the curve.
68-95-99.7: ~68% within 1 SD, ~95% within 2 SD, ~99.7% within 3 SD of the mean.
z-score = (x − mean) ÷ SD: how many SDs a value is from the mean; 80 → z = 2 here.
Ex: mean 70, SD 5 → ~95% of scores fall between 60 & 80; ~68% between 65 & 75.

⚠ A longer whisker ≠ more data; each quarter of a box plot holds ~25% of the values.

✓ Compare box plots by their medians (center) & box widths (IQR, the spread of data).

Bivariate Data

Scatterplots & Correlation

Association: positive (↗), negative (↘) or none; points close to a line → strong.
r is between −1 & 1: near ±1 → strong linear; near 0 → no linear link; sign = direction.

Line of Best Fit

Slope = predicted change in y for each 1-unit increase in x: ŷ = 2.5x + 40 → +2.5.
Predict: plug in x: x = 10 → ŷ = 2.5(10) + 40 = 65.
y-intercept = predicted y when x = 0; it may not make sense in context (e.g., age 0).
Residual = actual − predicted: 70 − 65 = +5 → the point is above the line (− → below).
Strong vs. weak: points hugging the line → strong; a loose cloud → weak; a curve → not linear.
Extrapolation far outside the data is risky.
Joint = cell ÷ grand total; **conditional** = cell ÷ its own row or column total only, not all.

Two-Way Tables

	Cats	Dogs	Total
10th	12	18	30
11th	20	10	30
Total	32	28	60

Ex: P(cats | 10th) = 12/30 = 0.4 (row total); P(10th & cats) = 12/60 = 0.2 (grand total).

⚠ Correlation ≠ causation: a strong r doesn't prove that x causes y; another factor may.

✓ "Given" group = the denominator.

Data Collection & Inference

Sampling

Random sample: everyone has an equal chance → results can apply to the population.
Bias: convenience or volunteer samples & leading questions skew the results of a survey.

Method	How
Simple random	Lottery from a list
Stratified	Random from each group
Systematic	Every kth person
Convenience	Whoever is nearby ⚠

Study Types

Observational study / survey: just measures → association only, not cause & effect.
Experiment: randomly assigned treatments → can show cause & effect; compare to a control.

Estimates

Sample → population: 30% of a sample → ~30% of 5,000 people = ~1,500 people.
Margin of error: 52% ± 3% → the true value is likely between 49% & 55% for everyone.
Bigger random sample → smaller margin of error.
Simulation: if a result rarely happens by chance (under ~5%), reject the claim being tested.

⚠ A big sample can still be biased if it isn't random; size doesn't fix a bad method.

✓ Random sampling → generalize; random assignment → show cause & effect.

Probability Basics

Core Ideas

P(event) = favorable ÷ total outcomes; always between 0 (impossible) & 1 (certain).
Complement: P(not A) = 1 − P(A).
Ex: 3 red, 5 blue, 2 green → P(blue) = 5/10 = ½; P(not green) = 8/10 = 4/5 of the time.

Sample Spaces

Experiment	Outcomes
1 coin	2
1 die	6
2 coins	4
2 dice	36
Deck of cards	52

2 dice: a sum of 7 happens 6 ways out of 36 → 6/36 = 1/6, the most likely sum.

Independent vs. Dependent

Independent: the 1st result doesn't change the 2nd (coin flips, w/ replacement).
Dependent: w/o replacement → 2 reds from 3R, 2B → 3/5 · 2/4 = 6/20 = 3/10 in all.
Experimental P = results ÷ trials: 18 heads in 40 flips → 0.45 (theoretical = 0.5).
Geometric P = target area ÷ total area: a target that's ¼ of the board → P = ¼ per throw.

⚠ No replacement → change the top & the bottom of the fraction on the 2nd pick.

✓ "At least one" → 1 − P(none).

Probability Rules

AND / OR Rules

Rule	Formula
AND (indep.)	P(A) · P(B)
OR	P(A) + P(B) − P(both)
OR (no overlap)	P(A) + P(B)

Mutually exclusive: can't happen together (roll a 2 & a 5 at once) → P(both) = 0; just add.
Overlap: P(heart or king) = 13/52 + 4/52 − 1/52 = 16/52 = 4/13 (king of hearts once).

Conditional Probability

P(A | B) = P(A and B) ÷ P(B): the chance of A given that B already happened ("|" = given).
From a table: use only the given row or column, then divide by that row or column total.
Independent if P(A | B) = P(A): knowing B doesn't change the chance of A happening at all.
Ex: 12 of the 40 athletes play an instrument → P(instrument | athlete) = 12/40 = 0.3.
Not A: P(not A) = 1 − P(A) (complement rule).

Venn Diagrams

Ex: 18 soccer, 12 band, 5 both → soccer or band = 18 + 12 − 5 = 25; of 30 students, 5 do neither.
Repeated events: P(3 heads in a row) = $(½)^3$ = 1/8.
Odds for = favorable : unfavorable; P = 3/8 → odds 3 : 5 (not 3 : 8); odds ≠ probability.

⚠ Don't count the overlap twice in OR problems.

✓ "Given" → shrink to that group first.

Counting: Permutations & Combinations

Counting Principle

Multiply choices: 3 shirts × 4 pants × 2 shoes = 24 different outfits in all (3·4·2).
Factorial: n! = n(n − 1)···1; 5! = 120 ways to line up 5 books in a row; 6! = 720; 0! = 1.
PIN: 4 digits, no repeats → 10·9·8·7 = 5,040; repeats allowed → 10^4 = 10,000.

Order Matters?

Type	Order?	Example
Permutation	Yes	Pres., VP, sec.
Combination	No	3-person team

Permutation: 3 officers from 10 people → 10·9·8 = 720 ways (roles mean order matters).
Combination: a 3-person team from 10 → 720 ÷ 3! = 120 teams (order doesn't matter).
Key words: arrange, rank, order → permutation; choose, select, team → combination.
Repeated letters: arrangements of BOOK = 4! ÷ 2! = 12 (the 2 O's look alike, so divide).

Probability by Counting

P = favorable selections ÷ total selections; use nCr on the top & on the bottom of the fraction.
Round table: n people → (n − 1)!; 5 people → 4! = 24 seatings (rotations count once).
At least 1 = total − none: pick 3 of 10 people w/ ≥ 1 of the 4 girls → 10C3 − 6C3 = 120 − 20 = 100.

⚠ Combinations ignore order → divide by r!.

✓ Use the nPr & nCr calculator keys.

Expected Value & Test Strategy

Expected Value

E(X) = sum of (value × probability): the long-run average result per trial, not a guarantee.

Outcome	Value	P	Value × P
Win	$50	0.1	$5
Small win	$10	0.3	$3
Lose	−$5	0.6	−$3

Ex: E = 5 + 3 − 3 = $5 per play; a $5 ticket → fair game; a $6 ticket → lose $1 a play.
Missing P: all P's add to 1 → missing P = 1 − (sum of the others): 0.2 & 0.5 → 0.3 left.
Expected count = n × p: 200 rolls of a die → 200 × 1/6 ≈ 33 sixes; 50 flips → 25 heads.
Negative E → you lose money on average; positive E → you win on average over time.

Test Format

Enhanced ACT math: 45 Qs (41 scored) in 50 min, 4 choices, calculator allowed throughout.
Stats & Probability: 5–6 scored Qs (12–15%).
No formula sheet: memorize mean, the probability rules, nPr, nCr & E(X) before test day.

Strategy

Read labels first: units, scale & totals on every table or graph before you compute.

⚠ A probability above 1 or below 0 is always wrong; cross those choices out.

✓ No guessing penalty → answer every question.