The Ultimate ACT® Enhanced Math Statistics and Probability Cheat Sheet

Center & Spread

Measures of Center

Mean = sum ÷ count: 4, 6, 8, 10, 22 → 50/5 = 10. **Median** = middle value once sorted → 8 (less than the mean).
Even count → median = avg of the 2 middle values: 3, 5, 9, 11 → (5 + 9)/2 = 7, even though 7 isn't in the set.
Mode = most frequent value; a set can have 0, 1 or several modes; it's the only center for category data.
Missing value: mean of 5 #s = 12 → sum = 60; 4 known #s sum to 47 → 5th # = 60 − 47 = 13. Work w/ sums.
Weighted mean: 20 students avg 80 + 30 avg 90 → (20·80 + 30·90)/50 = 86, not 85; bigger group pulls harder.
New mean after adding a value: (old sum + new value) ÷ (n + 1); 5 #s avg 10, add a 22 → 72/6 = 12 (mean ↑).
Which center? Symmetric data → mean; skewed data or outliers → median; category data → mode only.

Measures of Spread

Range = max − min. **IQR** = Q3 − Q1 (the middle 50%); IQR resists outliers, but the range doesn't at all.
Quartiles: Q1 = median of the lower half; Q3 = median of the upper half (leave out the median if n is odd).
Standard deviation (SD): typical distance from the mean; bigger SD → data more spread out; same units as data.

Changing the Data

Change	Mean / median	Range / IQR / SD
Add k to all	+ k	no change
Multiply all by k	× k	× |k|
Add an outlier	mean moves	range ↑, SD ↑

Outlier pulls the mean toward it; the median barely moves → median = better center for skewed data.

⚠ SD can't be negative; SD = 0 only when every value is the same (e.g., 5, 5, 5, 5); adding k never changes SD.

✓ Compare spreads w/ a picture: tightly clustered dots → smaller SD; widely spread dots → a bigger SD.

Distributions & Displays

Reading Displays

Display	Shows best
Dot plot	Each value, clusters
Histogram	Shape, bins of counts
Box plot	5-number summary
Bar chart	Categories, counts
Stem-and-leaf	Exact values, shape

5-number summary: min, Q1, median, Q3, max → box = IQR, line in the box = median, whiskers = min & max.
Outlier rule: below Q1 − 1.5·IQR or above Q3 + 1.5·IQR; shown as a dot past the whisker. Q1 10, Q3 20 → < −5 or > 35.
Histogram bins: read counts per interval; you can't find the exact mean from bins, only estimate it from midpoints.
Compare 2 box plots: compare medians for center & IQRs (box widths) for spread, not the whisker lengths.
Percentile: the 80th percentile → 80% of values are at or below it; it is not a score of 80% correct.

Shape

Symmetric: mean ≈ median. **Right-skewed** (tail right): mean > median. **Left-skewed**: mean < median.
Uniform: bars ≈ equal height. **Bimodal**: 2 peaks → often 2 groups mixed together (e.g., adults & kids).

Normal Distribution

Bell curve: symmetric; mean = median = mode at the center.
68-95-99.7 rule: ~68% within 1 SD, ~95% within 2 SD, ~99.7% within 3 SD of the mean in a normal curve.
Ex: mean 70, SD 5 → ~95% of scores between 60 & 80; ~2.5% above 80; ~16% above 75 (1 SD up).
z-score = (x − mean)/SD: x = 80 → z = 2; compares values across different sets (higher z = more unusual).

⚠ Box plots show quartiles, not counts: a longer whisker ≠ more data points; each quarter holds ~25% of the data.

✓ Skew rule: the mean is pulled toward the tail.

Bivariate Data

Scatterplots & Correlation

Association: positive (↗), negative (↘) or none; describe direction, form (linear or curved?) & strength (tight?).
r (correlation coefficient): −1 ≤ r ≤ 1; |r| near 1 → strong linear; r ≈ 0 → no linear link; sign = direction.
Strength check: points hugging the line → strong; loose cloud → weak; a curve → use a nonlinear model.

r	Meaning
0.9	Strong positive
0.4	Weak positive
−0.3	Weak negative
0	No linear pattern
−0.8	Strong negative

Line of Best Fit

Slope = predicted change in y per 1-unit ↑ in x: ŷ = 2.5x + 40 → +2.5 per unit of x (in the units of y).
y-intercept = predicted y when x = 0; may be meaningless if x = 0 is outside the data (e.g., a height of 0).
Predict: plug x into the line; x = 10 → ŷ = 2.5(10) + 40 = 65.
Residual = actual − predicted: actual 70, predicted 65 → +5 (the point lies above the line); − → below.
Extrapolation: predicting far outside the data's x-range is unreliable; the trend may not continue.
Influential point: an outlier far from the rest can change the slope & weaken r a lot; check if it's an error.

Two-Way Tables

Joint = cell ÷ grand total; **marginal** = row/column total ÷ grand total; both use the grand total as denominator.
Conditional = cell ÷ its row or column total: P(likes cats | 10th grade) = (cats & 10th) ÷ (all 10th graders).

⚠ Correlation ≠ causation: a strong r never proves that x causes y; a lurking variable may drive both.

✓ Read the "given" group: it's the denominator.

Data Collection & Inference

Sampling

Random sample: every member has an equal chance → results can generalize to the population.
Bias: convenience, voluntary response or leading questions → the sample isn't representative of the population.

Method	How
Simple random	Lottery from the list
Stratified	Random from each group
Cluster	Pick whole groups
Systematic	Every kth person
Convenience	Whoever is nearby ⚠

Study Types

Survey / observational: measure w/o intervening → shows association only, never cause & effect.
Experiment: treatments randomly assigned → can support cause & effect; use a control group for comparison.
Placebo & blinding: a fake treatment & hidden group labels keep expectations from skewing the results.
Generalize? Random sampling → to the population. **Cause?** Random assignment → treatment effect.

Estimates & Margin of Error

Sample → population: 30% of a 200-person sample → ~30% of 5,000 people = ~1,500 (scale the proportion).
Margin of error: 52% ± 3% → plausible true value 49%–55%; a bigger random sample → a smaller margin.
Sample size: 4× the sample → margin of error ½ as big (it shrinks w/ √n), so bigger random samples → more precise.
Simulation: if the observed result rarely happens by chance (e.g., < 5%), reject the claim; if common, don't.

⚠ A margin of error covers random sampling error only, not bias from a bad sample; bias can't be fixed by size.

✓ Overlapping intervals (e.g., 48–54% vs. 50–56%) → no clear difference between the 2 groups.

Probability Basics

Core Ideas

P(event) = favorable outcomes ÷ total equally likely outcomes; 0 ≤ P ≤ 1 (0 = impossible, 1 = certain).
Complement: P(not A) = 1 − P(A). "At least 1" → 1 − P(none).
Ex: bag w/ 3 red, 5 blue, 2 green → P(blue) = 5/10 = ½; P(not green) = 8/10 = 4/5; P(purple) = 0 (impossible).

Sample Spaces

Experiment	Outcomes	Ex: P
1 coin	2	P(H) = ½
1 die	6	P(even) = ½
2 coins	4	P(2 H) = ¼
2 dice	36	P(sum 7) = 1/6
Deck	52	P(heart) = ¼

List it: 2 dice → sum 7 in 6 ways: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) → 6/36 = 1/6, the most likely sum.

Independent vs. Dependent

Independent: the 1st result doesn't change the 2nd (coin flips, draws w/ replacement) → just multiply.
Dependent: w/o replacement → the total & counts drop: 2 reds from 3R, 2B → 3/5 · 2/4 = 6/20 = 3/10.
Geometric P = target area ÷ total area: circle r = 1 inside a 4 × 4 square → π/16 ≈ 0.20 (a dart lands in it).
Tree diagram: multiply along branches for AND, add the end results for OR; branches from 1 node sum to 1.

Expected vs. Actual

Experimental vs. theoretical: 18 heads in 40 flips → experimental P = 0.45; theoretical P = 0.5.
Law of large numbers: more trials → the experimental P gets closer to the theoretical P (not in the short run).

⚠ "Without replacement" → change BOTH the numerator & the denominator on the 2nd draw, every time.

✓ "At least one" → use the complement: 1 − P(none).

Probability Rules

AND / OR Rules

Rule	Formula	Use when
AND (indep.)	P(A)·P(B)	Both happen
AND (dep.)	P(A)·P(B|A)	No replacement
OR	P(A) + P(B) − P(A and B)	Either happens
OR (mut. ex.)	P(A) + P(B)	Can't overlap

Mutually exclusive: can't happen together (rolling a 2 & a 5 on 1 die) → P(A and B) = 0, so just add.
Overlap: P(heart or king) = 13/52 + 4/52 − 1/52 = 16/52 = 4/13 (subtract the king of hearts, counted twice).
Venn diagrams: 30 students, 18 soccer, 12 band, 5 both → soccer or band = 18 + 12 − 5 = 25; neither = 5.
Neither: P(neither A nor B) = 1 − P(A or B); **not both**: 1 − P(A and B); read the wording very carefully.

Conditional Probability

P(A | B) = P(A and B)/P(B): the probability of A given that B already happened; read "|" as "given."
From a table: restrict to the given row or column, then divide by that row or column total, not the grand total.
Ex: 40 of 100 students play sports; 12 of them play an instrument → P(instr. | sports) = 12/40 = 0.3.
Independence test: A & B are independent if P(A | B) = P(A), or if P(A and B) = P(A)·P(B); otherwise dependent.

Odds & Repeated Events

Odds for = favorable : unfavorable; P = 3/8 → odds 3 : 5 (not 3 : 8); odds 2 : 7 → P = 2/9 (add the parts).
Repeated trials: P(3 heads in a row) = (½)³ = 1/8; P(no 6 in 2 rolls) = (5/6)² = 25/36 ≈ 0.69 (about 69%).

⚠ Mutually exclusive ≠ independent: if A happens, B can't, so they DEPEND on each other completely.

✓ "Given" = shrink the sample space to that group first.

Counting: Permutations & Combinations

Counting Principle

Multiply choices: 3 shirts × 4 pants × 2 shoes = 24 outfits.
Factorial: n! = n(n − 1)···1; 5! = 120; 0! = 1. Arrange 5 books in a row → 5! = 120 ways; 6 books → 720.
Restrictions: 4-digit PIN, no repeats → 10·9·8·7 = 5,040; w/ repeats → 10^4 = 10,000; fill fixed spots first.
Together as a block: 2 friends must sit side by side → 4! · 2! = 48 ways to line up 5 people (block + swap).

Order Matters?

Type	Order?	Formula
Permutation	Yes	n!/(n − r)!
Combination	No	n!/(r!(n − r)!)

Permutation: pres., VP, sec. from 10 people → 10·9·8 = 720 (distinct roles make the order matter).
Combination: choose a 3-person committee from 10 → 10C3 = 720/3! = 120 (same 3 people in any order = 1).
Key words: arrange, rank, order, assign roles → permutation; choose, select, group, team → combination.
Repeated letters: arrangements of BOOK = 4!/2! = 12; divide by the factorial of each repeat count (O ×2).
At least 1 = total − none: 3 from 6 boys & 4 girls w/ ≥ 1 girl → 10C3 − 6C3 = 120 − 20 = 100 possible teams.

Probability by Counting

P = (# favorable selections) ÷ (# total selections): pick 2 of 5 people, Ana included → 4C1/5C2 = 4/10 = 0.4.
Ex: 3 cards from 52, all hearts → 13C3/52C3 = 286/22,100 ≈ 0.013; same as 13/52 · 12/51 · 11/50 multiplied.
Circular seating of n people → (n − 1)!: 5 people at a round table → 4! = 24 (rotations count as the same).

⚠ Combination counts ≤ permutation counts b/c order is ignored: nCr = nPr ÷ r! (10P3 = 720 → 10C3 = 120).

✓ Calculator: use the nPr & nCr keys to save time on big numbers; check your setup w/ a small case.

Expected Value & Test Strategy

Expected Value

E(X) = Σ (value × probability): the long-run average outcome per trial; weight each value by its chance.

Outcome	Value	P	Value × P
Win big	$50	0.1	$5
Win small	$10	0.3	$3
Lose	−$5	0.6	−$3

Ex: E = 5 + 3 − 3 = $5 per play; a game costing $5 to play would be fair (E − cost = 0); cost $6 → lose $1/play.
Probability distribution: all P's sum to 1; a missing P = 1 − (sum of the others): 0.2, 0.5, ? → 0.3 here.
Counts: expected # = n·p: 200 rolls of a die → 200 · 1/6 ≈ 33 sixes expected; 50 coin flips → 25 heads.
Games & insurance: net E = Σ(payoff × P) − cost; a negative E → the player loses money on average.
Mean of a distribution = E(X): values 0, 1, 2 w/ P = 0.2, 0.5, 0.3 → 0(0.2) + 1(0.5) + 2(0.3) = 1.1 on average.

Test Format

Enhanced ACT math: 45 Qs (41 scored, 4 unscored field tests) in 50 min, 4 choices, calculator allowed.
Stats & Probability = 5–6 scored Qs (12–15%); basic stats & data also appear in Integrating Essential Skills.
No formula sheet: memorize mean, P rules, nPr, nCr & E(X).

Strategy

Read the table/graph labels first: units, scale, totals, and which variable sits on each axis of the graph.
Check reasonableness: a probability > 1 or < 0, or a negative count, is always wrong; cut those choices.

⚠ "Expected value" can be a value no single trial can produce (e.g., 3.5 for 1 roll of a fair 6-sided die).

✓ No guessing penalty → answer every Q; flag long data sets & come back to them after the quick Qs.