The Simple AP® Calculus AB Unit 1 Cheat Sheet

1.1–1.4 What a Limit Means

1.1–1.2 Idea & Notation

Avg rate = [f(b) − f(a)]/(b − a) (secant slope).
Instant rate = its limit as interval → 0.
$\lim_{x\to c} f(x) = L$: f(x) → L as x → c (both sides).

One-sided: $x\to c^-$ (from left), $x\to c^+$ (from right).
Two-sided limit exists only if left = right.
f(c) irrelevant: can be missing or ≠ L.

1.3 From Graphs

Graph at x = c	Limit?
Hole	y of the hole
Jump	DNE
Vertical asymptote	±∞ (DNE)
Oscillates	DNE

Trace toward c from both sides; skip dot.

1.4 From Tables

Tables: watch outputs as x → c from both sides; a table is only an estimate, not proof.
e.g. x = 0.99 → 1.99; x = 1.01 → 2.01 → limit ≈ 2.
e.g. hole at (2, 3) + filled dot at (2, 5) → $\lim_{x\to 2} f$ = 3, but f(2) = 5, so f is not continuous at 2.
Tables can mislead: a few sample points may miss a jump or a spike right near x = c.

Graph Checklist

Check: left = right? → limit. f(c) = dot.
⚠ Open/closed dot changes f(c), not limit.
✓ Hole + dot: limit = hole y; f(c) = dot y.

1.5–1.6 Limit Laws & Algebra

1.5 Limit Properties

Limit laws: if both limits exist, limits split over +, −, ×, ÷ (denom limit ≠ 0) and over powers.

Law	Result
Sum	L + M
Product	L·M
Quotient	L/M (M ≠ 0)
Constant	k·L

Direct substitution: plug in c first; works for polynomials & most nice functions.
Composition: lim f(g(x)) = f(lim g(x)) if f is continuous at that inner limit value.

1.6 Algebra Tricks

0/0 (indeterminate) → simplify, then plug in again.
Factor & cancel: $(x^2 - 9)/(x - 3) = (x - 3)(x + 3)/(x - 3) = x + 3 \to 3 + 3 = 6$ at x = 3.
Conjugate: for √ terms, multiply top & bottom by the conjugate (√ + #), then cancel the zero factor.
Common denom: combine stacked fractions into one fraction, cancel, then plug in.
Expand: $[(2 + h)^2 - 4]/h = 4 + h \to 4$.
Difference of squares: $a^2 - b^2 = (a - b)(a + b)$.
Cubes: $(x^3 - 8)/(x - 2) = x^2 + 2x + 4 \to 12$.
e.g. $(\sqrt{(x + 4)} - 2)/x \to x/[x(\sqrt{(x + 4)} + 2)] \to 1/(\sqrt{(x + 4)} + 2) \to 1/(2 + 2) = ¼$ at x = 0.
Preview: $\lim_{h\to 0} [f(a + h) - f(a)]/h = f'(a)$ (Unit 2).

⚠ Canceling fixes the 0/0, not the hole: f(c) is still undefined, so f is still discontinuous.
✓ Show the algebra step on FRQs.

1.7–1.8 Strategy & Squeeze Theorem

1.7 Choose a Procedure

Plug in, get	Do this
A number	that's the limit
0/0	factor, conjugate
k/0 (k ≠ 0)	check signs → ±∞
Piecewise break	check both sides

Absolute value: $|x - c|/(x - c) = 1$ from the right, −1 from the left → limit DNE (a jump at c).
Piecewise: use the formula for each side of c, then compare the 2 one-sided limits you get.
Flow: plug in → number? done. 0/0? algebra. Stuck? squeeze, graph or a table.

1.8 Squeeze Theorem

If g ≤ f ≤ h near c and g, h → same L, then f → L.
Classic: $-x^2 \le x^2\sin(1/x) \le x^2$ → limit 0 at x = 0.
Bound trig: −1 ≤ sin, cos ≤ 1, then multiply through.
e.g. $-|x| \le x\cdot\cos(1/x) \le |x|$ → limit 0 at x = 0.

Special Trig Limits

$\lim_{x\to 0} \sin x / x = 1$ (radians only, never degrees).

$\lim_{x\to 0} (1 - \cos x)/x = 0$ (radians).

Scale: sin(5x)/x → 5; sin(kx)/x → k.
More: tan x / x → 1; sin(3x)/sin(2x) → 3/2.
Trap: sin x / x → 1 only as x → 0; as x → ∞ it → 0.

⚠ Both bounds must share the same limit.
✓ FRQ: write the inequality, find both bound limits, then state the conclusion for f.

1.9–1.10 Representations & Discontinuities

1.9 Multiple Representations

Same limit can be shown by a graph, table, formula or words; FRQs mix them.
Words: "f(x) approaches L as x approaches c" = $\lim_{x\to c} f(x) = L$; translate both ways.

Tables estimate; graphs & formulas confirm.
Tables alone can't prove continuity unless the stem says f is continuous (or gives a formula).

1.10 Types of Discontinuity

Type	Limit at c
Removable (hole)	exists
Jump	left ≠ right
Infinite (VA)	±∞

Removable: hole, or a point moved off the curve.
Jump: often where piecewise pieces don't meet.
Infinite: denominator factor that doesn't cancel.
e.g. left → 2, right → 2, f(c) = 5: removable.
Continuous ≠ smooth: |x| is continuous at 0 but has a sharp corner there (no derivative).

Spot It from a Formula

Factor fully: canceled factor → hole; leftover denominator factor → VA; leftover top → zero.
e.g. $(x - 2)/(x^2 - 4) = 1/(x + 2)$ for x ≠ 2: hole at (2, ¼), VA at x = −2, and no x-intercept.

⚠ "f(c) undefined" alone doesn't tell you the type of break: always check the limit at c.
✓ Name the type + give limit values as evidence.

1.11–1.13 Continuity

1.11 Continuity at a Point

f is continuous at c if all 3 are true:

Condition	Check
1. f(c) defined	plug in
2. limit exists	left = right
3. limit = f(c)	compare

1.12 Over an Interval

Open (a, b): continuous at every point inside.
Closed [a, b]: continuous inside + one-sided limits at a (from right) & b (from left).
Continuous on domain: polynomials, rational, roots, exp, log & trig (sums & products too).
Combos: sums, products & compositions of continuous functions stay continuous.

1.13 Removing Discontinuities

Removable: define f(c) = the limit.
Piecewise w/ k: set both pieces equal at the break point (plug in c), then solve for k.
e.g. kx + 1 (x < 2), x^2 (x ≥ 2): 2k + 1 = 4 → k = 3/2.
e.g. $f = (x^2 - 1)/(x - 1)$ for x ≠ 1, f(1) = k: the limit is 2, so k = 2 makes f continuous at x = 1.
Differentiable ⇒ continuous (not the reverse).
Typical FRQ: "Is f continuous at x = 3? Justify." → find both one-sided limits + f(3), compare.

⚠ FRQ: show all 3 conditions w/ values.
✓ Write: left limit = right limit = f(c) = __ (show the values), so f is continuous at x = c.

1.14 Infinite Limits & Vertical Asymptotes

Infinite Limits

lim f = ∞: unbounded growth (DNE).
k/0 form (k ≠ 0): check sign on each side.
e.g. 1/(x − 3): right → +∞, left → −∞ → DNE.
e.g. $1/(x - 3)^2$: both sides → +∞.
Indeterminate: ∞ − ∞, 0·∞, ∞/∞.

Vertical Asymptotes

x = c is a VA if a one-sided limit at c is ±∞.

Function	VA at
rational	denom zeros left
ln x	x = 0
tan x	x = π/2 + nπ

Sign Test Steps

Steps: factor → test x just L/R of c → sign.
Even power in denom → same sign both sides.
e.g. $(x + 1)/(x - 2)$ at 2^+: (+)/(small +) → +∞.

Worked Example

$(x^2 - x - 2)/(x^2 - 4)$ = (x + 1)/(x + 2), x ≠ 2.
x = 2: cancels → hole at (2, ¾). **x = −2**: VA.

ln x → −∞ as $x \to 0^+$, so VA at x = 0.
Odd power at VA → the 2 sides split (+∞, −∞); **even** → both go the same way.
Calculators hide holes: confirm w/ algebra.

⚠ Canceled denom zero = hole, NOT VA.
✓ Justify a VA w/ a one-sided limit.

1.15 Limits at Infinity & HAs

Rational Functions: Degree Rule

Degrees	lim at ±∞
top < bottom	0
top = bottom	lead coeff ratio
top > bottom	±∞ (no HA)

Method: divide every term by the highest power of x in the denominator, then let x → ∞.
e.g. $(3x^2 - 1)/(5x^2 + x) \to 3/5$ (same degree).
e.g. $(4x + 7)/(x^2 + 1) \to 0$ (bottom wins).

Roots & Both Directions

$\sqrt{(x^2)} = |x|$: as x → −∞, $\sqrt{(x^2)} = -x$.
e.g. $2x/\sqrt{(x^2 + 1)}$: → 2 (x→∞), → −2 (x→−∞).

Other Functions

e^x: → 0 as x → −∞; grows fast as x → ∞.

Growth order: ln x < x^n < e^x (for large x).
HA: y = L if f → L as x → ∞ or as x → −∞. A graph is allowed to cross its own HA, too.
arctan x → π/2 (x→∞), −π/2 (x→−∞).
At most 2 HAs: one each direction.

Worked Example

(6x + 1)/(2x − 4): same degree → HA y = 6/2 = 3; denom zero, no cancel → VA at x = 2.
Why: the top powers dominate for big |x|, so all smaller-power terms shrink toward 0.

⚠ Check x → −∞ too: roots, e^x.
✓ HA = end behavior only, not near-c behavior.

1.16 IVT & Exam Moves

Intermediate Value Theorem

If f is continuous on [a, b] and k is between f(a) and f(b) → some c in (a, b) has f(c) = k.
Root version: f(a), f(b) have opposite signs → f has a zero somewhere in (a, b) (if f is continuous).
Existence only: IVT doesn't find c.
Doesn't say how many: could be several c's.

IVT Justification

1. State f is continuous on [a, b]. **2.** f(a) < k < f(b). **3.** So by IVT, f(c) = k for some c in (a, b).

Given	IVT gives
f(1) = −2, f(4) = 3	zero in (1, 4)
f not continuous	no conclusion

Table FRQ: f cont., f(2) = 5, f(6) = −1, f(9) = 4 → 2 sign changes → at least 2 zeros in (2, 9).
EVT preview: f continuous on a closed [a, b] → abs. max & min both exist there (Unit 5).
Trick: to show f(c) = c, use IVT on g(x) = f(x) − x.

Unit 1 on the Exam

MCQ: limits from graphs/tables, 0/0 algebra, discontinuity types, finding k values.
FRQ: continuity of piecewise f, IVT from tables.
Format (May 2027): MC 42 Qs (29 no calc + 13 calc); FRQ 6 Qs (2 calc + 4 no calc).
Notation: keep writing "lim" until you plug in.

⚠ No continuity statement = no IVT point.
✓ Name the theorem + check its conditions.