The Ultimate AP® Calculus AB Unit 1 Cheat Sheet

1.1–1.4 What a Limit Means

1.1 Change at an Instant

Avg rate = [f(b) − f(a)]/(b − a) = secant slope. **Instantaneous rate** = limit of avg rates as interval → 0 (tangent slope).

1.2 Notation

$\lim_{x\to c} f(x) = L$: f(x) gets arbitrarily close to L as x → c from both sides; x never has to equal c, and f(c) can be anything.

One-sided: $x\to c^-$ (from left), $x\to c^+$ (from right).

Two-sided limit exists ⇔ left limit = right limit (both finite).

f(c) is irrelevant: limit can exist w/ f(c) undefined or ≠ L.

1.3 From Graphs

Trace along the curve from each side toward x = c; read the y-value approached, never the dot sitting at c.

Graph at x = c	Limit?
Hole, curve meets	exists = y of hole
Jump	DNE (L≠R)
Vertical asymptote	±∞ or DNE
Oscillates (sin 1/x)	DNE

1.4 From Tables

Tables: watch outputs as x → c from both sides (e.g., 0.9, 0.99, 0.999); a table can estimate, never prove.

Tables can mislead: a few sample points may miss a jump, a spike or wild oscillation right near x = c.

e.g. $(x^2 - 1)/(x - 1)$: x = 0.9, 0.99 → 1.9, 1.99; x = 1.01 → 2.01 → limit ≈ 2, even though f(1) is undefined (0/0).

Graph Reading Checklist

1. Left limit. **2.** Right limit. **3.** Equal? → two-sided limit exists. **4.** Then f(c) separately (filled dot, if any).

Composite lim f(g(x)): find lim g = M + the side g approaches from → use that one-sided limit of f at M.

⚠ Open circle vs. closed dot changes f(c), never the limit.

✓ MCQ: when a graph has a hole + a separate dot, the limit = the hole's y-value; f(c) = the dot's y-value.

1.5–1.6 Limit Laws & Algebra

1.5 Limit Properties

If lim f = L and lim g = M (both finite), then:

Law	Result
Sum/diff	L ± M
Constant multiple	k·L
Product	L·M
Quotient	L/M (M ≠ 0)
Power/root	L^n, $\sqrt[n]{L}$
Composition	f(lim g) if f cont. at M

Direct substitution: works for polynomials, rational (denom ≠ 0), trig, exp, log, roots on their domains.

1.6 Algebraic Manipulation

0/0 = indeterminate → rework, don't stop.

Factor & cancel: $(x^2 - 9)/(x - 3) = x + 3$ → limit at 3 is 6.

Conjugate: $(\sqrt{x+4} - 2)/x \cdot (\sqrt{x+4} + 2)/(\sqrt{x+4} + 2) = x/[x(\sqrt{x+4} + 2)] = 1/(\sqrt{x+4} + 2) \to ¼$.

Common denom: combine complex fractions, then cancel.

Expand: $[(2 + h)^2 - 4]/h = (4h + h^2)/h = 4 + h \to 4$.

Trig identities: $\sin^2 x + \cos^2 x = 1$, double-angle to simplify.

Complex fraction: $(1/x - 1/3)/(x - 3) = (3 - x)/[3x(x - 3)] = -1/(3x) \to -1/9$ at x = 3 (multiply top & bottom by 3x).

Cubes: $(x^3 - 8)/(x - 2) = x^2 + 2x + 4 \to 12$.

Factoring Kit

$\mathbf{a^2 - b^2} = (a - b)(a + b)$. $\mathbf{a^3 \pm b^3} = (a \pm b)(a^2 \mp ab + b^2)$.

Trinomials: $x^2 - x - 6 = (x - 3)(x + 2)$. Factor out GCF first.

Laws need parts to exist: lim(f + g) can exist even when lim f & lim g don't (e.g., jumps that cancel); then use algebra.

Derivative preview: $\lim_{h\to 0} [f(a + h) - f(a)]/h = f'(a)$ (Unit 2).

⚠ Canceling a factor removes the 0/0, not the hole: f(c) is still undefined in the original f, so f stays discontinuous.

✓ Show the algebra step on FRQs (factor, conjugate, cancel): a bare final answer may lose the point.

1.7–1.8 Strategy & Squeeze Theorem

1.7 Choose a Procedure

Substitution gives	Do this
Number L	done: limit = L
0/0	factor, conj., trig ID
k/0 (k ≠ 0)	check signs → ±∞/DNE
∞/∞ (x → ∞)	compare degrees
Piecewise at break	check both sides

Absolute value: $|x - c|/(x - c) = 1$ from the right, −1 from the left → two-sided limit DNE (jump at c).

Piecewise: use the formula valid on each side of c; compare.

Calculator (Part B): table w/ tiny steps or a zoomed graph to confirm a limit found by algebra; never as proof.

1.8 Squeeze Theorem

If $g(x) \le f(x) \le h(x)$ near c and lim g = lim h = L → lim f = L.

Classic: $-x^2 \le x^2\sin(1/x) \le x^2$ → limit at 0 is 0.

Bound trig: $-1 \le \sin, \cos \le 1$; multiply through, watch sign.

e.g. $x\cdot\cos(1/x)$: $-|x| \le x\cdot\cos(1/x) \le |x| \to 0$ at x = 0.

Special Trig Limits

$\lim_{x\to 0} \sin x / x = 1$ (x in radians only; degrees break it).

$\lim_{x\to 0} (1 - \cos x)/x = 0$ (also radians only).

Scale: $\sin(5x)/x = 5\cdot[\sin(5x)/(5x)] \to 5$. $\sin(3x)/\sin(2x) \to 3/2$.

More: $\tan x / x \to 1$; $x / \sin x \to 1$; $\sin^2 x / x \to 0$.

Conj. trick: $(1 - \cos x)/x^2 \cdot (1 + \cos x)/(1 + \cos x) = \sin^2 x/[x^2(1 + \cos x)] = (\sin x/x)^2 \cdot 1/(1 + \cos x) \to 1 \cdot ½ = ½$.

Trap: $\sin x / x \to 1$ only as x → 0; as x → ∞ it → 0.

Flow: plug in → number? done. 0/0? algebra or trig ID. Still stuck? squeeze, or graph/table to estimate.

⚠ Squeeze needs BOTH bounds to share the same limit L; state the full inequality first, then take limits.

✓ FRQ squeeze: write the inequality, find both bound limits, then state the conclusion for f by the Squeeze Thm.

1.9–1.10 Representations & Discontinuities

1.9 Multiple Representations

Connect graph, table, formula & words: same limit, different evidence; FRQs mix 2+ of them in 1 part.

Words: "as x approaches c, f(x) approaches L" = $\lim_{x\to c} f(x) = L$; translate both ways on MCQs & FRQ parts.

Table + graph: table estimates; graph/formula confirms.

Tables alone can't prove continuity: you need "f is continuous" given in the stem, or a formula to check.

VA in words: "vertical asymptote at x = 1" ⇒ $\lim_{x\to 1^+} f$ or $\lim_{x\to 1^-} f = \pm\infty$ (just 1 side is enough for a VA).

1.10 Types of Discontinuity

Type	Limit at c	Fix?
Removable	exists	redefine f(c)
Jump	L ≠ R (finite)	no
Infinite	±∞ (VA)	no

Removable: hole, or point moved off curve (lim ≠ f(c)).

Jump: common in piecewise functions w/ mismatched pieces at the break; both one-sided limits finite.

Infinite: factor in denominator that doesn't cancel.

Oscillating (e.g., sin(1/x) at 0): no limit; not removable.

e.g. left → 2, right → 2, f(c) = 5: removable (lim 2 ≠ f(c)).

Step functions: ⌊x⌋ jumps at every integer.

Continuous ≠ smooth: |x| is continuous at 0 but has a corner there (not differentiable at 0; see Unit 2).

Spot It from a Formula

Rational f: factor fully. Canceled factor → hole; leftover denom factor → VA; numerator zero → x-int.

e.g. $(x - 2)/(x^2 - 4) = 1/(x + 2)$ for x ≠ 2: hole at x = 2 (y = ¼), VA at x = −2, no x-intercept (num ≠ 0 after cancel).

Piecewise f: test each break point only; each piece is already continuous on its own open interval (if nice).

⚠ "f(c) undefined" alone doesn't tell you the type: find the limit to tell a hole vs. a VA vs. a jump.

✓ Name the type + give the limit values as evidence.

1.11–1.13 Continuity

1.11 Continuity at a Point

f continuous at x = c iff all 3 hold (check in order):

Condition	Check
1. f(c) defined	plug in
2. $\lim_{x\to c} f(x)$ exists	left = right
3. lim = f(c)	compare

1.12 Over an Interval

Open (a, b): continuous at every point inside.

Closed [a, b]: + one-sided at ends (a^+, b^-).

Cont. on domain: poly, rational, root, exp, log, trig.

Combos: sums, products, compositions of cont. functions stay cont. (quotients where denom ≠ 0).

tan x: discontinuous at π/2 + nπ (VAs) but still continuous on its domain; same idea for 1/x at x = 0 (not in its domain).

1.13 Removing Discontinuities

Removable: define f(c) = $\lim_{x\to c} f(x)$ to fill the hole.

Piecewise w/ k: left formula(c) = right formula(c) → k.

e.g. f = kx + 1 (x < 2), x^2 (x ≥ 2): 2k + 1 = 4 → k = 3/2.

e.g. f = $(x^2 - 1)/(x - 1)$ for x ≠ 1, f(1) = k: factor → x + 1 → limit 2, so k = 2 makes f continuous at x = 1 (removable fixed).

Composition: g cont. at c & f cont. at g(c) → f(g(x)) cont. at c; e.g., $\sqrt{x^2 + 1}$ and $\sin(x^2)$ are cont. everywhere.

e.g. f = sin x / x (x ≠ 0), f(0) = a: a = 1.

2 unknowns: cont. → 1 eq.; diff. → 2nd (Unit 2).

Differentiable ⇒ continuous (converse false).

Typical FRQ: "Is f continuous at x = 3? Justify." → find both one-sided limits & f(3); show all 3 are equal.

⚠ FRQ: show all 3 conditions w/ values.

✓ Write: $\lim_{x\to c} f = f(c) = __$ → cont. at c.

1.14 Infinite Limits & Vertical Asymptotes

Infinite Limits

$\lim_{x\to c} f(x) = \infty$: f grows w/o bound near c. Limit technically DNE; "= ∞" just describes how it fails to exist.

k/0 form (k ≠ 0): sign test on each side of c.

e.g. 1/(x − 3): $x\to 3^+ \to +\infty$; $x\to 3^- \to -\infty$ → two-sided DNE.

e.g. $1/(x - 3)^2$: both sides → +∞, so lim = ∞.

Arithmetic: k/∞ = 0; k·∞ = ±∞ (k ≠ 0); ∞ + ∞ = ∞. **∞ − ∞, 0·∞, ∞/∞, 0/0** = indeterminate → rework.

Vertical Asymptotes

x = c is a VA if either one-sided limit at c is ±∞.

Function	VA at
rational p/q	zeros of q not canceled
ln x	x = 0 (from right)
tan x, sec x	x = π/2 + nπ
cot x, csc x	x = nπ

Sign Test Steps

1. Factor. **2.** Plug a value just left/right of c (e.g., c ± 0.01). **3.** Sign of num ÷ sign of denom → +∞ or −∞ on that side.

e.g. (x + 1)/(x − 2) at 2^+: (+)/(small +) → +∞.

Even power in denom → same sign both sides (one infinite limit); **odd** → signs flip (two-sided limit DNE).

Graphs hide holes: graphing calculators often skip them; confirm any hole or VA algebraically by factoring.

Worked Example

$f = (x^2 - x - 2)/(x^2 - 4)$ = (x − 2)(x + 1)/[(x − 2)(x + 2)].

x = 2: factor cancels → hole at (2, ¾). **x = −2**: VA.

$x\to -2^+$: $(-)/(0^+) \to -\infty$. **$x\to -2^-$**: $(-)/(0^-) \to +\infty$.

Others: $\lim_{x\to 0^+} \ln x = -\infty$ (VA x = 0).

Shape: odd power at VA → ends split; even → same way.

⚠ Zero of the denominator that cancels = hole, NOT a VA.

✓ Justify a VA w/ a one-sided limit, not "denom = 0."

1.15 Limits at Infinity & HAs

Rational Functions: Degree Rule

Degrees	$\lim_{x\to\pm\infty}$
top < bottom	0 (HA y = 0)
top = bottom	ratio of lead coeffs
top > bottom	±∞ (no HA)

Method: divide every term by highest power of x in denom.

e.g. $(3x^2 - 1)/(5x^2 + x) \to 3/5$ (equal degrees).

e.g. $(4x + 7)/(x^2 + 1) \to 0$ (HA y = 0). $(2x^3 + 1)/(x^2 - 5) \to \infty$ as $x \to \infty$ and $\to -\infty$ as $x \to -\infty$ (no HA at all).

Roots & Both Directions

$\sqrt{x^2} = |x|$: for $x \to -\infty$, $\sqrt{x^2} = -x$ (sign flip).

e.g. $2x/\sqrt{x^2 + 1}$: $x\to\infty \to 2$; $x\to-\infty \to -2$ (two HAs).

Conjugate at ∞: $\sqrt{x^2 + x} - x = x/(\sqrt{x^2 + x} + x) \to ½$.

Other Functions

e^x: → 0 as $x \to -\infty$; → ∞ as $x \to \infty$ (HA y = 0 on left).

Growth order: $\ln x \ll x^n \ll e^x$. So $x^2/e^x \to 0$, $\ln x / x \to 0$.

Squeeze at ∞: $-1/x \le \sin x / x \le 1/x \to 0$.

arctan x → π/2 (x→∞), −π/2 (x→−∞). **e^{-x}** → 0 as $x \to \infty$.

Logistic $10/(1 + e^{-x})$: HAs y = 10 (x→∞) and y = 0 (x→−∞).

HA: y = L if $\lim_{x\to\infty}$ or $\lim_{x\to-\infty} f = L$. Graph CAN cross an HA.

At most 2 HAs (one per direction). **Top deg = bottom + 1** → slant asymptote, no HA in either direction.

Calculator (Part B): table at $x = 10^3, 10^6, -10^6$ to confirm end behavior in each direction before answering.

Worked Example

f = (6x + 1)/(2x − 4): equal degrees → HA y = 6/2 = 3.

VA x = 2: $x\to 2^+$: $13/(0^+) \to +\infty$; $x\to 2^- \to -\infty$.

Why it works: highest powers dominate as |x| grows, so all lower-degree terms → 0 by comparison.

⚠ Check $x \to -\infty$ separately for roots, exponentials, arctan.

✓ HA = end behavior ($x \to \pm\infty$) only, not near c.

1.16 IVT & Exam Moves

Intermediate Value Theorem

If f continuous on [a, b] and k is between f(a) and f(b) → ∃ c in (a, b) w/ f(c) = k. Both conditions needed.

Root version: f(a), f(b) opposite signs → f has a zero in (a, b).

Existence only: IVT doesn't find c or say how many.

Trick: to show f(c) = c, apply IVT to g(x) = f(x) − x.

EVT preview: f continuous on a closed [a, b] → abs. max & abs. min both exist on [a, b] (Unit 5 uses this).

IVT Justification Template

1. f is continuous on [a, b] (state why: differentiable, polynomial, or given in the stem; cite it).

2. f(a) = __ < k < __ = f(b) (or the reverse order).

3. So by IVT, ∃ c in (a, b) w/ f(c) = k. Done.

Given data	IVT gives
f(1) = −2, f(4) = 3	zero in (1, 4)
cont. + 2 sign flips	≥ 2 zeros
f not continuous	no conclusion

Table FRQ: f cont., f(2) = 5, f(6) = −1, f(9) = 4 → 2 sign changes → f = 0 at least twice in (2, 9), once per interval.

Unit 1 on the Exam

MCQ: limits from graphs/tables, 0/0 algebra, discontinuity types, finding k for continuity, squeeze setups.

FRQ: continuity of a piecewise f, IVT from tables; limit ideas feed into every later unit (2–8), esp. derivatives.

Format (May 2027): MC 42 Qs: 29 no calc (62 min) + 13 calc (38 min). FRQ 6 Qs: 2 calc (30 min) + 4 no calc (60 min). MC = 50%, FRQ = 50%; graphing calc on calc parts.

Notation: keep writing "lim" in every line until the step where you substitute; dropping it early costs points.

⚠ IVT w/o stating continuity on the closed interval [a, b] = no point, even w/ the right c and correct values.

✓ Use exact theorem names (IVT, Squeeze, EVT) + verify every hypothesis in writing before the conclusion.