The Simple AP® Calculus AB Unit 2 Cheat Sheet

2.1 Average vs. Instantaneous Rate

Average Rate of Change

Avg rate on [a, b] = [f(b) − f(a)]/(b − a) = slope of the **secant line** (thru 2 points on f).

Ex: f(x) = x² on [1, 3] → (9 − 1)/(3 − 1) = 4. Units: output per input, e.g., ft/sec, gal/min.

Instantaneous Rate of Change

Inst. rate at a = **tangent** slope = f′(a).

Secant → tangent: as the interval shrinks (h → 0), secant slopes → the tangent slope.

Average rate	Instantaneous rate
Over an interval	At 1 point
Secant slope	Tangent slope
Algebra only	Needs a limit
Avg. velocity	Velocity v(t)

In Context

Position s(t): avg velocity = Δs/Δt over an interval; velocity at 1 instant v(t) = s′(t), in m/sec.

Acceleration a(t) = v′(t) = rate of change of v.

Interpret f′(3) = 2: "At t = 3 hr, water level ↑ at 2 cm/hr." Give time, value & units.

Sign: rate > 0 → increasing; rate < 0 → decreasing; rate = 0 → momentarily constant.

Speed = |velocity| ≥ 0; velocity's sign = direction.

⚠ Avg rate of change ≠ avg of values. Use Δoutput/Δinput, not [f(a) + f(b)]/2.

✓ FRQ "approximate the rate" from a table → use a difference quotient & show it.

2.2 Definition of the Derivative

Limit Definitions

f′(a) = $\lim_{h\to 0}$ [f(a + h) − f(a)]/h, if the limit exists (same from both sides); else DNE.

Alt. form: f′(a) = $\lim_{x\to a}$ [f(x) − f(a)]/(x − a); same value, x replaces a + h; here x ≠ a.

Recognize the Limit

Pattern: $\lim_{h\to 0}$ [(2 + h)³ − 8]/h = f′(2) for f(x) = x³ → 3(2)² = 12; don't expand (2 + h)³.

By algebra: [(x + h)² − x²]/h = 2x + h → 2x.

Pattern: $\lim_{x\to\pi}$ (cos x + 1)/(x − π) = f′(π) → 0.

Notation

Notation	Read as
f′(x)	f prime of x
dy/dx	Derivative of y w.r.t. x
d/dx [f(x)]	Derivative of f(x)
y′	y prime
$dy/dx\big\|_{x=a}$	Value at x = a

Tangent Line Approximation

Tangent line at a: y = f(a) + f′(a)(x − a).

Ex: f = √x at 4: f′(x) = 1/(2√x) → f′(4) = 1/4.

Local linearization: f(x) ≈ tangent line near a. Ex: √4.1 ≈ 2 + (1/4)(0.1) = 2.025 (≈ true).

Units of f′: output per input, e.g., gal/min.

⚠ f′(a) = a NUMBER (slope); f′(x) = a FUNCTION. Differentiate 1st, then plug in a.

2.3 Estimating Derivatives

From a Table

Estimate f′(a) w/ the 2 closest points on either side of a: [f(b) − f(c)]/(b − c), c < a < b.

t (min)	0	2	5	9
W(t) (°F)	55	57	62	70

Ex: W′(3.5) ≈ (62 − 57)/(5 − 2) = 5/3 °F/min.

Symmetric difference: f′(a) ≈ [f(a + h) − f(a − h)]/2h, centered at a; usually the best estimate.

One-sided: [f(a + h) − f(a)]/h when data exists on only 1 side of a (e.g., the 1st t value).

Interpret: "At t = 3.5 min, temp ↑ at ≈ 1.667 °F per min." Give time, units & direction (↑/↓).

From a Graph of f

Tangent slope: draw tangent at a, rise/run ≈ f′(a).

Sketch f′ from f: f ↑ → f′ > 0; f ↓ → f′ < 0; peaks & valleys of f → f′ = 0 (f′ crosses the axis).

Line pieces: straight segment of f → f′ is constant (a horizontal piece on f′'s graph).

Steeper f → bigger |f′|; flat → f′ = 0.

Read f′'s graph: f′ > 0 → f ↑; f′ < 0 → f ↓; f′ = 0 → f has a flat (horizontal) tangent there.

With a Calculator

nDeriv / d/dx: f′(a) to 3 decimals on calc parts.

Local linearity: zoom in on a smooth graph → it looks like a line w/ slope f′(a) at that point.

⚠ Use the 2 values that straddle a.

✓ Show the setup (62 − 57)/(5 − 2); a bare decimal gets no FRQ credit, even if right.

2.4 Differentiability & Continuity

The Key Theorem

Differentiable ⇒ continuous: if f′(a) exists, f is continuous at a; a smooth curve has no breaks.

Converse FALSE: |x| is continuous at 0, no f′(0).

Contrapositive: not continuous at a (jump, hole, asymptote) ⇒ no f′(a) there, ever.

Where f′ Fails to Exist

Feature	Example	Why
Corner	|x| at 0	Slopes differ
Cusp	$x^{2/3}$ at 0	Slopes → ±∞
Vertical tangent	$\sqrt[3]{x}$ at 0	Slope infinite
Discontinuity	Jump, hole	Not continuous

Piecewise Functions

Step 1: pieces meet (left lim = right lim = f(a)). **Step 2**: slopes match at a (left f′ = right f′).

Ex: f = x² (x ≤ 1), kx + m (x > 1). Meet: 1 = k + m. Slopes: 2 = k → so k = 2, m = −1.

Vertical tangent: $\sqrt[3]{x}$ has f′ → ∞ at 0.

Reading Graphs

Scan the graph for sharp points, vertical tangents, breaks & holes; elsewhere f′ exists.

Endpoints: theorems need "differentiable on the open interval (a, b)" only, not at a & b.

⚠ "Continuous, so differentiable" is a classic wrong answer; corners are continuous.

✓ To justify "not differentiable," show the 1-sided slopes differ or f isn't continuous.

2.5–2.6 Power Rule & Basic Rules

Power Rule

Power rule: d/dx [x^n] = nx^{n-1} for any constant n; e.g., (x^5)′ = $5x^4$, (x^{-1})′ = $-x^{-2}$ = $-1/x^2$.

Rewrite	Then	Derivative
$\sqrt{x} = x^{1/2}$	$(1/2)x^{-1/2}$	$1/(2\sqrt{x})$
$1/x^3 = x^{-3}$	$-3x^{-4}$	$-3/x^4$
$\sqrt[3]{x^2} = x^{2/3}$	$(2/3)x^{-1/3}$	$2/(3\sqrt[3]{x})$
$5/x = 5x^{-1}$	$-5x^{-2}$	$-5/x^2$

Rewrite 1st: √x → $x^{1/2}$; $1/x^n$ → x^{-n}.

Ex: y = 6√x − 2/x → y′ = 3/√x + 2/x².

Constant, Sum & Multiple

Rules: (c)′ = 0; (cf)′ = cf′; (f ± g)′ = f′ ± g′.

Ex: y = 4x³ − 2x + 7 → y′ = 12x² − 2.

Simplify 1st: (x² + 3x)/x = x + 3 → y′ = 1.

Higher-order: f″ = (f′)′: x⁴ → 4x³ → 12x².

Tangent & Normal Lines

Normal line (⊥ tangent): slope −1/f′(a).

Horizontal tangent: solve f′(x) = 0. f = x³ − 3x → 3x² − 3 = 0 → x = ±1 → (1, −2), (−1, 2).

Ex: tangent to y = x³ at x = 2: point (2, 8), slope 3(2)² = 12 → so y = 8 + 12(x − 2).

Given slope: y = x², slope 6 → 2x = 6 → (3, 9).

⚠ d/dx [2^x] ≠ $x \cdot 2^{x-1}$; the power rule needs a constant exponent; it's actually 2^x ln 2.

2.7 Derivatives of sin, cos, e^x & ln x

Core Derivatives

f(x)	f′(x)	Note
sin x	cos x	Radians only
cos x	−sin x	Watch the minus
e^x	e^x	Its own derivative
ln x	1/x	x > 0

a^x: d/dx [10^x] = 10^x ln 10 (extra factor ln a).

Radians only: set calc to radian mode.

Why They Work

Special limit: $\lim_{h\to 0}$ (sin h)/h = 1 → used to derive (sin x)′ = cos x from the limit definition.

Graph check: sin x has slope 1 at 0 (cos 0 = 1) & slope 0 at its peaks, where cos x = 0.

e^x: slope = height → (e^x)′ = e^x always.

Examples

Mix: 3 sin x − 2e^x + 5 ln x → 3 cos x − 2e^x + 5/x.

Ex: y = 4 cos x − x → y′ = −4 sin x − 1.

Log rules 1st: ln(x^3) = 3 ln x → 3/x; ln(5x) = ln 5 + ln x → 1/x (ln 5 is a constant, → 0).

Slopes: sin at π/3 → 1/2; ln at 2 → 1/2.

Ex: y = 3e^x + ln x at x = 1: slope 3e + 1 ≈ 9.15.

Tangent to e^x at 0: slope 1 → y = x + 1.

⚠ (cos x)′ = −sin x; don't drop the minus.

✓ $\lim_{h\to 0}$ [sin(π/6 + h) − sin(π/6)]/h = (sin x)′ at π/6 = cos(π/6) = √3/2; no algebra.

2.8–2.9 Product & Quotient Rules

Product Rule

Product: (fg)′ = f′g + fg′ ("1st′ · 2nd + 1st · 2nd′"). Ex: (x^2 sin x)′ = 2x sin x + x^2 cos x.

Ex: (xe^x)′ = e^x + xe^x = e^x(1 + x).

Quotient Rule

Quotient: (f/g)′ = (f′g − fg′)/g^2, g ≠ 0: "low d-high − high d-low, over low²"; keep g^2 as is.

Ex: (sin x/x)′ = (x cos x − sin x)/x^2.

Ex: (e^x/x)′ = (xe^x − e^x)/x^2 = e^x(x − 1)/x^2.

Order matters: f′g − fg′ ≠ fg′ − f′g; start w/ the derivative of the top; a sign flip = trap.

Tables of Values

x	f	f′	g	g′
2	3	−1	4	5
5	−2	3	1	−4

h = fg: h′(2) = (−1)(4) + (3)(5) = 11.

k = f/g: k′(2) = [(−1)(4) − (3)(5)]/4² = −19/16.

p = x·f(x): p′(5) = f(5) + 5f′(5) = −2 + 15 = 13.

q = g/x: q′(5) = [5g′(5) − g(5)]/5² = −21/25.

Tangent to fg at 5: point (5, −2), slope (3)(1) + (−2)(−4) = 11 → y + 2 = 11(x − 5) is the line.

From graphs: read f(a), g(a) off the curves; f′(a), g′(a) = slopes of their line pieces.

Skip the rule: (x^2 + 1)/5 → 2x/5.

⚠ (fg)′ ≠ f′g′ & (f/g)′ ≠ f′/g′; split sums only.

✓ Write the rule w/ letters, then plug in.

2.10 Other Trig Derivatives

Tan, Cot, Sec & Csc

f(x)	f′(x)
tan x	$\sec^2 x$
cot x	$-\csc^2 x$
sec x	sec x tan x
csc x	−csc x cot x

Co- rule: cos, cot, csc ("co-") all have NEGATIVE derivatives; tan & sec → positive ones.

Derive w/ Quotient Rule

tan x = sin x/cos x → (cos²x + sin²x)/cos²x = 1/cos²x = sec²x, using sin²x + cos²x = 1.

Domains: tan & sec are undefined where cos x = 0 (x = π/2 + kπ) → no derivative there.

Unit Circle Values

x	sin	cos	tan
0	0	1	0
π/6	1/2	√3/2	1/√3
π/4	√2/2	√2/2	1
π/3	√3/2	1/2	√3
π/2	1	0	Undef.

Ex: slope of tan x at π/4 = $\sec^2(\pi/4)$ = 2.

Ex: (5 sec x − cot x)′ = 5 sec x tan x + $\csc^2 x$.

Mix: (x tan x)′ = tan x + x $\sec^2 x$.

⚠ $\sec^2 x = (\sec x)^2$, not $\sec(x^2)$.

✓ Memorize all 6 trig derivatives & unit circle values; the exam gives no formula sheet.