The Simple AP® Calculus AB Unit 3 Cheat Sheet

3.1 The Chain Rule

The Rule

Chain rule: $d/dx[f(g(x))] = f'(g(x))\cdot g'(x)$: outer derivative (inside kept) × inner derivative.
Leibniz form: $dy/dx = (dy/du)\cdot(du/dx)$; $y = u^3$, $u = 2x + 1 \to 3u^2\cdot 2 = 6(2x + 1)^2$; the du's cancel.
Spot it: anything that isn't plain x is inside: $(3x + 1)^5$, $\sin(x^2)$, e^{2x}, $\ln(x^2 + 1)$, $\sqrt{(4 - x)}$.

Common Chain Forms

Function	Derivative
$[g(x)]^n$	$n[g(x)]^{n-1}\cdot g'(x)$
$\sqrt{g(x)}$	$g'(x)/(2\sqrt{g(x)})$
$e^{g(x)}$	$e^{g(x)}\cdot g'(x)$
$\ln g(x)$	$g'(x)/g(x)$
$\sin g(x)$	$\cos g(x)\cdot g'(x)$
$\cos g(x)$	$-\sin g(x)\cdot g'(x)$

Worked Examples

Power: $(3x + 1)^5 \to 5(3x + 1)^4\cdot 3 = 15(3x + 1)^4$; the ·3 is $d/dx(3x + 1)$, easy to forget.
Trig: $\sin(x^2) \to \cos(x^2)\cdot 2x$. **Exp**: $e^{-4x} \to -4e^{-4x}$.
Log: $\ln(x^2 + 1) \to 2x/(x^2 + 1) = \text{in}'/\text{in}$.
Power of trig: $\cos^4 x = (\cos x)^4 \to 4\cos^3 x\cdot(-\sin x)$; the 4th power is the outermost layer.

⚠ Keep the inside unchanged in f': $d/dx \sin(3x) = 3\cos(3x)$, not $\cos(3)\cdot 3$ or $\cos(x)\cdot 3$.

✓ Work outside → in: peel 1 layer at a time & multiply by each inner derivative.

3.1 Chain Rule: Tables, Graphs & Layers

From a Table

Setup: $h(x) = f(g(x)) \to h'(a) = f'(g(a))\cdot g'(a)$; find g(a) first, then f' at THAT value.

x	f(x)	f'(x)	g(x)	g'(x)
1	3	−2	2	4
2	5	6	1	−3
3	4	1	3	2

Ex: $h(x) = f(g(x)) \to h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot 4 = 6\cdot 4 = 24$; g from row 1, f' from row 2.
Ex: $k(x) = g(f(x)) \to k'(2) = g'(f(2))\cdot f'(2) = g'(5)\cdot 6 \to$ need g'(5); read every column carefully.

From a Graph

Slopes: f'(a) = slope of f's graph at x = a; on a line segment, slope = rise/run of that segment.
Corner: if g(a) lands on a corner or cusp of f, f'(g(a)) doesn't exist → h'(a) DNE (unless g'(a) = 0).
Graph + table mix: read f' from the graph's slope & g, g' from the table; label each source.

Multiple Layers & Mixed Rules

3 layers: $\sin^3(2x) = [\sin(2x)]^3 \to 3\sin^2(2x)\cdot\cos(2x)\cdot 2 = 6\sin^2(2x)\cos(2x)$.
Product + chain: $x^2e^{3x} \to 2xe^{3x} + 3x^2e^{3x} = xe^{3x}(2 + 3x)$; the 3 comes from the inner (3x)'.

⚠ $f'(g(a)) \neq f'(a)\cdot g(a)$: evaluate g first, then f' at g(a), then multiply by g'(a) at the end.

✓ Write $h'(a) = f'(g(a))\cdot g'(a)$ before plugging in; label each value you read from the table.

3.2 Implicit Differentiation

When & Why

Implicit: x & y mixed ($x^2 + y^2 = 25$, $xy + y^3 = 5$) → differentiate both sides w/o solving for y first.
Key: y is a function of x, so every y term gets a dy/dx by the chain rule: $d/dx(y^3) = 3y^2\cdot dy/dx$.

Steps

1. d/dx both sides. **2.** Add dy/dx to each y-derivative. **3.** Gather dy/dx terms, factor, divide.

Term	d/dx
y^2	$2y\cdot y'$
xy	$y + x\cdot y'$
$\sin y$	$\cos y\cdot y'$
e^y	$e^y\cdot y'$
$\ln y$	y'/y

Worked Examples

Circle: $x^2 + y^2 = 25 \to 2x + 2y\cdot y' = 0 \to y' = -x/y$; at (3, 4): slope = −3/4; at (3, −4): +3/4.
Product: $xy + y^3 = 10 \to y + xy' + 3y^2y' = 0 \to y' = -y/(x + 3y^2)$; factor y' out, then divide.

Horizontal & Vertical Tangents

Horizontal: numerator of dy/dx = 0. **Vertical**: denominator = 0 (other part ≠ 0).
Circle check: $y' = -x/y \to$ horizontal where x = 0 → (0, ±5); vertical where y = 0 → (±5, 0).

⚠ d/dx(xy) needs the PRODUCT rule: $y + x\cdot y'$, not just y'; same for x^2y and $x\cdot\sin y$.

✓ dy/dx may contain x & y → plug in BOTH coordinates of the point, not just x.

3.2 Implicit: Second Derivatives & FRQ Moves

Finding d^2y/dx^2

Method: differentiate dy/dx again (quotient rule), then substitute dy/dx back in & simplify.
Circle: $y' = -x/y \to y'' = -(y - x\cdot y')/y^2 = -(y + x^2/y)/y^2 = -(x^2 + y^2)/y^3 = -25/y^3$.
Use the curve: $x^2 + y^2 = 25$ turned the whole numerator into 25; look for that move.
At (3, 4): $y'' = -25/64 < 0 \to$ concave down there; at (3, −4): +25/64 → concave up.

Typical FRQ Parts

Part asks	Move
Show dy/dx = ...	Implicit diff + algebra
Tangent line at pt	Plug both coords → slope
Horizontal tangent	Numerator = 0 + check
Vertical tangent	Denominator = 0
Concavity at pt	Find y'', plug in

Horizontal tangent check: set numerator = 0, then plug back into the curve to find real points.
Tangent line ex: $x^2 + xy + y^2 = 7$ at (1, 2): $2x + y + xy' + 2yy' = 0 \to 4 + 5y' = 0 \to y' = -4/5$.
Normal line: slope = −1/(dy/dx); at (3, 4) on $x^2 + y^2 = 25$ the normal slope is 4/3 (flip & negate).
"Show that" parts: write every step; the given answer earns nothing w/o your work.

⚠ Don't drop y' inside y'': every y term still needs the chain rule when you differentiate again.

✓ Substitute the given point as early as allowed to save algebra (but find y'' symbolically first).

3.3 Derivatives of Inverse Functions

The Formula

Inverse derivative: $(f^{-1})'(a) = 1/f'(f^{-1}(a))$: the inverse's slope = reciprocal slope.
Reflection: if (b, a) is on f, then (a, b) is on f^{-1} & the slopes are reciprocals (m ↔ 1/m).
Why: $f(f^{-1}(x)) = x \to$ chain rule → $f'(f^{-1}(x))\cdot(f^{-1})'(x) = 1$; then solve for $(f^{-1})'(x)$ to get the formula.

Steps

1. Find $b = f^{-1}(a)$: solve f(b) = a (a table or a guess). **2.** Find f'(b). **3.** Answer is 1/f'(b).

x	f(x)	f'(x)
1	2	4
2	7	9
3	15	10

Table ex: $(f^{-1})'(7) \to f(2) = 7 \to f'(2) = 9 \to (f^{-1})'(7) = 1/9$; find 7 in the f(x) column, not x.
Formula ex: $f(x) = x^3 + x$, find $(f^{-1})'(2)$: $f(1) = 2 \to f'(1) = 4 \to$ answer 1/4 (no algebra).
Graph method: $(f^{-1})'(a) = 1/$(slope of f where y = a); read the tangent line slope off the graph.
Existence: needs f one-to-one near the point & f'(b) ≠ 0 (else f^{-1} has a vertical tangent there).

⚠ $(f^{-1})'(a) \neq 1/f'(a)$: evaluate f' at $f^{-1}(a)$, the x-value that gives a, not at a itself.

✓ First find which x makes f(x) = a; it's usually a small integer, a table entry or a quick guess.

3.4 Inverse Trig Derivatives

Memorize These

Function	Derivative
$\arcsin x$	$1/\sqrt{(1 - x^2)}$
$\arccos x$	$-1/\sqrt{(1 - x^2)}$
$\arctan x$	$1/(1 + x^2)$
$\text{arccot } x$	$-1/(1 + x^2)$
$\text{arcsec } x$	$1/(\lvert x\rvert\sqrt{(x^2 - 1)})$

Notation: $\arcsin x = \sin^{-1}x$, NOT $1/\sin x = \csc x$.
Co-functions: arccos, arccot & arccsc derivatives are the negatives of arcsin, arctan & arcsec.
Most tested: arcsin & arctan with the chain rule; arccos is just the negative of arcsin's derivative.

With the Chain Rule

Function	Derivative
$\arcsin u$	$u'/\sqrt{(1 - u^2)}$
$\arctan u$	$u'/(1 + u^2)$

Ex: $\arctan(3x) \to 3/(1 + 9x^2)$. **Ex**: $\arcsin(x^2) \to 2x/\sqrt{(1 - x^4)}$; the u' always goes on top.
Tangent line ex: $y = \arctan x$ at $x = 1 \to$ the point $(1, \pi/4)$, slope = ½ → $y - \pi/4 = \frac{1}{2}(x - 1)$.

Values to Know

Outputs: $\arcsin \frac{1}{2} = \pi/6$; $\arccos \frac{1}{2} = \pi/3$; $\arctan 1 = \pi/4$; $\arctan\sqrt{3} = \pi/3$; $\arcsin 1 = \pi/2$.

⚠ Square the WHOLE inner function (u = 2x): $\arctan(2x) \to 2/(1 + 4x^2)$, not $2/(1 + 2x^2)$.

✓ Inverse trig derivatives are algebraic (no trig left): a quick way to eliminate choices.

3.5 Selecting the Right Procedure

Pick the Rule

Expression looks like	Use
Sum/difference	Term by term
f·g	Product rule
f/g	Quotient (or rewrite)
Function inside function	Chain rule
x & y mixed	Implicit diff
f^{-1} at a point	$1/f'(f^{-1}(a))$

Simplify First

Rewrite: $5/x^3 = 5x^{-3} \to -15x^{-4}$; skip the quotient rule when the bottom is a constant or x^n.
Log rules: $\ln[(x^2)(x + 1)^3] = 2\ln x + 3\ln(x + 1) \to 2/x + 3/(x + 1)$; no product rule needed.

Order of Rules

Outermost first: $(x \sin x)^3 \to$ chain (cube) first, then product inside: $3(x \sin x)^2(\sin x + x\cos x)$.

Common Traps

Constants: $d/dx(e^2) = 0$ (a constant). $d/dx(x^e) = ex^{e-1}$ (power rule). $d/dx(e^x) = e^x$.
Other bases: $d/dx\, b^x = b^x\cdot\ln b$, so $2^x \to 2^x\cdot\ln 2$; $d/dx \log_b x = 1/(x\cdot\ln b)$; $\ln x$ is $b = e$.

⚠ Product rule ≠ product of derivatives: $(fg)' = f'g + fg'$, never just f' times g' alone.

✓ Name the outermost operation out loud; it tells you the first rule to apply, then work inward.

3.6 Higher-Order Derivatives & Exam Moves

Notation

2nd derivative: $f''(x) = y'' = d^2y/dx^2$; just differentiate f' again (and again for the 3rd, 4th, ...).
Ex: $f(x) = x^4 - 3x^2 \to f' = 4x^3 - 6x \to f'' = 12x^2 - 6 \to f''' = 24x \to f^{(4)} = 24 \to f^{(5)} = 0$.

Patterns

f(x)	Pattern
$\sin x$	$\sin \to \cos \to -\sin \to -\cos$
e^{kx}	$f^{(n)} = k^ne^{kx}$
$\ln x$	$1/x$, $-1/x^2$, $2/x^3$ ...
x^n (whole #)	0 after n + 1 derivatives

Ex: $d^{27}/dx^{27}(\sin x)$: $27 = 4\cdot 6 + 3 \to$ same as the 3rd derivative → $-\cos x$ (cycle of 4).

Meaning

f'' = rate of change of f': $f'' > 0 \to$ f' increasing → concave up; $f'' < 0 \to$ concave down.
Motion preview: position s → velocity s' → acceleration s'' (used heavily in Unit 4).

Exam Format

MCQ: Unit 3 ≈ 9–13% of AB MCQs; **FRQ**: chain, implicit & inverse derivatives appear inside many.

⚠ f'' is the derivative of f', not $(f')^2$; don't square the 1st derivative to get the 2nd one.

✓ No-calc section: show the chain-rule factor; unsimplified correct answers still earn credit.