The Ultimate AP® Physics 1 Unit 1 Cheat Sheet

1.1 Scalars & Vectors in 1D

Scalar vs. Vector

Scalar: magnitude only. **Vector**: magnitude + direction; drawn as an arrow w/ length ∝ magnitude.

Quantity	Type	Units
Distance	Scalar	m
Speed	Scalar	m/s
Time	Scalar	s
Position x	Vector	m
Displacement Δx	Vector	m
Velocity v	Vector	m/s
Acceleration a	Vector	m/s²

Signs = Direction (1D)

Choose + first (right/up usually) and state it. In 1D the sign of a component fully gives its direction; no arrow needed.
1D vector sum: add signed values. 5 m right + 8 m left = (+5) + (−8) = −3 m → 3 m to the left of start.
Magnitude = |value| ≥ 0. −3 m/s = 3 m/s in the − direction, not a speed "less than zero" or a slower one.
Negative ≠ slowing: a sign only gives direction. Slowing ↔ v and a have opposite signs, whichever way they point.

Distance vs. Displacement

Distance: total path length (always +). **Displacement**: $\Delta x = x_f - x_i$ (can be +, − or 0; the path doesn't matter).

Round trip: back to the starting point → Δx = 0 but distance > 0 → avg velocity = 0, while avg speed > 0.

FRQ Moves

Compare claims: state magnitude AND direction ("A's v is larger in magnitude, opposite direction").

⚠ Switching sign convention mid-problem is the #1 kinematics error. Pick + once; use it for every vector.

✓ Mark + on a number line before any equation.

1.2 Displacement, Velocity & Acceleration

Object Model

Object model: ignore size, shape, internal parts → treat as 1 point w/ mass. (Rotation waits for Unit 5.)

Average vs. Instantaneous

Avg velocity: $v_{avg} = \Delta x/\Delta t$ (displacement, NOT distance).

Avg acceleration: $a_{avg} = \Delta v/\Delta t$. Only the initial and final velocities matter, not what happens in between.

Instantaneous: average over a tiny Δt ≈ instantaneous value (slope of the tangent line at that instant).

Avg ≠ mean of speeds: equal distances at 10 then 30 m/s → v_{avg} = 15 m/s, not 20 (more time spent slow).

Speeding Up or Slowing Down?

Accelerating ↔ speed OR direction of v changes. Constant speed on a curve still accelerates (direction changes).

v sign	a sign	Motion
+	+	speeding up, +
+	−	slowing, moving +
−	−	speeding up, −
−	+	slowing, moving −
0	≠ 0	turning around

v = 0 ≠ a = 0: top of a toss: v = 0, a = −g.

Worked Examples

Car: +20 → +8 m/s in 3 s: a = (8 − 20)/3 = −4 m/s² → slowing down (v +, a −); a points backward, opposite v.
Bounce: +10 → −10 m/s off a wall in 0.1 s: Δv = −20 m/s, a = −200 m/s². Same speed before & after, but Δv ≠ 0.

⚠ $\Delta v = v_f - v_i$ WITH signs. A bounce doubles Δv; never subtract the two magnitudes (10 − 10 ≠ Δv).

✓ Sign check: v and a same sign → speed ↑; opposite signs → speed ↓; v = 0, a ≠ 0 → turnaround.

1.3 Kinematic Equations (Constant a)

The 3 Sheet Equations

$v_x = v_{x0} + a_x t$: no Δx; use when position isn't given or asked.

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$: no final v; position at time t.

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$: no t; best for stopping.

Derived: $\Delta x = \frac{1}{2}(v_0 + v)t$ (no a), const a only.

Valid ONLY for constant a. If a changes, split into stages; final v of stage 1 = initial v of stage 2.

Pick the Equation

Missing / unwanted	Use
Δx	$v = v_o + at$
final v	$\Delta x = v_o t + \frac{1}{2}at^2$
t	$v^2 = v_o^2 + 2a\Delta x$
a	$\Delta x = \frac{1}{2}(v_o + v)t$

Steps: sketch → pick + → list Δx, v_0, v, a, t → need 3 knowns → pick the eqn w/o the unwanted variable.

Symbolic Derivations (MR/QQT)

Derive: start from a sheet equation, substitute, solve for the target. Answer only in given symbols + physical constants; numbers last. Check units & limiting cases.
Stopping distance: $0 = v_0^2 - 2ad \rightarrow d = v_0^2/(2a)$. 2× v_0 → 4× d; 2× braking a → ½ d. Speed matters most.
Scaling: from rest, $\Delta x \propto t^2$ → 2× time = 4× distance; $v \propto t$.
Multi-stage: 0 → 20 m/s in 4 s, then coast 6 s: Δx = ½(20)(4) + 20(6) = 40 + 120 = 160 m total (area under v–t).
Catch-up: same x at same t → set $x_1(t) = x_2(t)$ and solve for t.

⚠ Hidden knowns: "from rest" or "dropped" → $v_o = 0$; "comes to a stop" or "at the peak" → v = 0.

✓ "What happens if v_o doubles?" Write the eqn, hold the other quantities fixed, then scale the answer.

1.3 Motion Graphs: Slope & Area

Slope & Area Rules

Graph	Slope =	Area =
x vs. t	velocity	—
v vs. t	acceleration	displacement
a vs. t	—	Δv

Instant v = slope of the tangent on x–t. **Avg v** = slope of the secant line between 2 points on the curve.
Area below the t-axis counts negative for displacement; add the |areas| for the total distance traveled.

Read the Shape

Shape	On x–t	On v–t
Horizontal	at rest	const v (a = 0)
Straight slope	const v	const a
Curves up	a > 0	a increasing
Curves down	a < 0	a decreasing
Crosses t-axis	passes x = 0	turns around

Turnaround: v–t crosses zero; x–t has a peak or valley (slope 0) at that same t. a need not be 0 there (a toss).
Speeding up on v–t: line moves AWAY from the t-axis.
Slowing: line moves toward the t-axis (|v| ↓).

Sketching (TBR FRQ)

Translate: x–t → v–t: plot each segment's slope. v–t → a–t: plot its slope; const a = flat horizontal line.
Nonuniform a: AP1 asks only qualitatively: sketch the curve's shape & explain it; no calculation needed.
Sketch rules: label axes w/ quantity + units, mark key times (v = 0), keep x and v continuous (no jumps).

⚠ v–t height ≠ position. Highest point on v–t = max velocity, not the point farthest from the start (x).

✓ Justify w/ the rule: "slope of x–t is v; slope of v–t is a; area under v–t is Δx," then apply it to the graph.

1.3 Free Fall & Motion Diagrams

Free Fall

Near Earth: a = g, downward, constant. Use g = 10 m/s² (9.8 also accepted). Ignore air unless told.
Up = +: $a_y = -10$ m/s² the WHOLE flight: while rising, at the very peak & while falling back down to the ground.
Peak: v = 0, a = −g. Time up = time down when launch & landing heights match (no air resistance on AP1).
Symmetry: returns to launch height w/ the same speed, in the opposite direction; same speed at each height.
Mass doesn't matter: heavy and light objects fall w/ the same a (no air resistance): a = g for both.
Thrown down at v_0: a is still −g; it just starts w/ $v_0 < 0$ (up = +), so its speed grows the whole way down to the ground.
Drop from a rising balloon: object starts w/ the balloon's upward v, rises briefly, then falls (a = −g).

Toss up at 20 m/s	Result
Time to peak	v_o/g = 2 s
Max height	$v_o^2/(2g)$ = 20 m
Total air time	4 s
Speed on return	20 m/s, downward

Motion Diagrams

Dot diagram: dots at equal Δt. Equal spacing → const v; growing → speeding up; shrinking → slowing.
Velocity arrows: 1 per dot, length ∝ speed. **a arrow**: direction of Δv ($v_i \rightarrow v_f$), not of v itself; draw it once, beside the dots.

Nonuniform Acceleration

Air resistance: a falling object's a ↓ as speed ↑ → v–t curve flattens toward terminal speed. Qualitative only.

⚠ At the peak a ≠ 0. Gravity never switches off; only v is 0, and only for an instant before it falls.

✓ Use up-down symmetry to halve the work: solve the way up, then double the time for the full flight.

1.4 Reference Frames & Relative Motion

Reference Frames

Reference frame: an observer's viewpoint + axes. The frame sets the measured direction AND magnitude of x and v.
Inertial frame: not accelerating. Assume every frame is inertial (constant v) unless told otherwise.

Relative Velocity (1D only)

Rule: observed v = object's v in the frame + frame's v. Subscripts: $v_{AG} = v_{AB} + v_{BG}$ (inner letters cancel).

Reverse it: $v_{BA} = -v_{AB}$. Signs carry direction.

Situation	Observed v
Walk +1 on train +20	+21 m/s (ground)
Walk −1 on train +20	+19 m/s (ground)
Cars +25, +30 m/s	5 m/s apart
Cars +25, −30 m/s	55 m/s closing
Train seen from car +30	−10 m/s if train +20

Moving walkway: time = length/($v_{walk} \pm v_{belt}$); use + w/ the belt, − against it. Walking w/ the belt is faster.
Gap closing: time = gap ÷ relative speed (100 m gap, 5 m/s closing speed → 20 s until the 2 cars meet).

Acceleration Is Frame-Free

Same a in all inertial frames: a ball dropped on a steady train has a = g for the rider AND the ground observer.
Different paths: rider sees a straight drop; ground sees a parabola (adds the train's v). Both are right.
Position: also frame-dependent; the gap between two frames' x readings grows by $v_{rel}\Delta t$ over time.

Toss in a steady elevator: rider sees an ordinary toss; ground adds the elevator's v. Same a = −g.

⚠ AP1 keeps relative-velocity math to 1D. Expect add/subtract along a line, not 2D river-crossing triangles.

✓ Write the subscript chain first; the outer letters must match what the Q asks for (e.g., v_{AG}).

1.5 Vectors & Components in 2D

Components

Resolve: $A_x = A\cos\theta$, $A_y = A\sin\theta$ (θ from the +x axis). Trig + Pythagorean rules are on the equation sheet.

Rebuild: $A = \sqrt{(A_x^2 + A_y^2)}$; $\theta = \tan^{-1}(A_y/A_x)$.

Signs by direction: pointing left → $A_x < 0$; down → $A_y < 0$.

Draw it; don't trust the calculator's angle.

θ	cos θ	sin θ
0°	1	0
30°	0.87	0.50
45°	0.71	0.71
60°	0.50	0.87
90°	0	1

Angle from vertical? Swap: $A_x = A\sin\theta$, $A_y = A\cos\theta$. cos always goes w/ the side next to θ (adjacent).

Adding Vectors

Tip-to-tail: resultant runs from the 1st tail to the last tip.
Components: add x's, add y's, rebuild.
Size limits: $|A - B| \le |A + B| \le A + B$. Perpendicular vectors → use the Pythagorean theorem to find the sum.

2D displacement: 3 m E then 4 m N → 5 m at $\tan^{-1}(4/3) \approx 53°$ N of E; distance walked = 3 + 4 = 7 m.

Independence of x & y

Key idea: perpendicular components don't affect each other → 2D motion = two 1D problems sharing the same t.
2D velocity: $v = \sqrt{(v_x^2 + v_y^2)}$; direction from $\tan\theta = v_y/v_x$.
Each component is ≤ the magnitude of v.

⚠ Calculator in DEGREE mode. A sin/cos mix-up flips which component is bigger; sketch to check.

✓ Sanity check: at 30° the x-component is bigger; at 60° the y-component is bigger than x; at 45° they're equal.

1.5 Projectile Motion

The Model

Projectile: only gravity acts → $a_x = 0$, $a_y = -g$. x: constant velocity; y: free fall. Both share the same time t.
Launch: $v_{x0} = v_0\cos\theta$, $v_{y0} = v_0\sin\theta$. Time t links x and y.

	x	y
Accel.	0	−g
Velocity	constant	changes
Equation	$x = v_x t$	sheet eqns
At peak	v_x same	$v_y = 0$

Horizontal launch: $v_{y0} = 0 \rightarrow t = \sqrt{(2h/g)}$; range = $v_x t$.

Drop vs. throw: dropped and horizontally launched balls from the same height land at the SAME time.
Level-ground range: max at 45°; complementary angles (30° & 60°) give the same range, different heights.
Speed anywhere = $\sqrt{(v_x^2 + v_y^2)}$; min at peak = v_x.

Max height: $H = v_{y0}^2/(2g)$. **Level flight time**: $t = 2v_{y0}/g$. Both depend only on v_{y0}, never on the horizontal v_x.

Ex: 20 m/s at 30°: $v_x \approx 17.3$ m/s, $v_{y0} = 10$ m/s → t = 2 s, H = 5 m, range ≈ 17.3 × 2 ≈ 35 m on level ground.

FRQ Moves

Justify: name the principle + tie it to the scenario: "only gravity acts → v_x const → equal horizontal spacing."

LAB: linearize. From rest, plot Δx vs. t^2 → straight line, slope = ½a. Use the best-fit slope, not 1 data point.

⚠ Fall time depends only on vertical motion (h, v_{y0}, g), never on the horizontal speed v_x. Faster ≠ longer.

✓ Solve the vertical motion first to get t, then use that same t for the horizontal motion ($x = v_x t$) to find range.