The Ultimate AP® Physics 2 Unit 10 Cheat Sheet

10.1 Charge & Coulomb's Law

Charge Basics

Charge: + or −; smallest unit e = 1.6×10^{-19} C. Electron −e, proton +e, neutron 0. All charge = whole multiples of e.

Point charge: size negligible vs. distances in the problem.

Coulomb's Law: Ratio Reasoning

$F \propto q_1q_2/r^2$: direct w/ each charge, inverse-square w/ distance. Line of action = line joining charges.

Change	New F_E
$q_1 \times 2$	2F
$q_1 \times 2$, $q_2 \times 3$	6F
$r \times 2$	F/4
$r \times 1/3$	9F
both q × 2, r × 2	F (no change)

Like signs repel; opposite attract. Sign tells direction only: use $|q|$ in the magnitude, then draw the arrow.

Newton's 3rd law: F on q_1 = F on q_2 in size, even if $q_1 \gg q_2$. Accelerations differ if masses differ (a = F/m).

Net Force (≤4 Charges)

Superposition: find each pair force separately → add as vectors (x & y components), never as plain numbers.

Zero-force spot: between like charges (closer to smaller q); outside opposite charges (beyond smaller q).

Equilateral triangle, 3 equal +q: two forces F at 60° → F_{net} = √3F, pointing away from the triangle's center.

Units: k = 9.0×10^{9} N·m²/C²; μC → $\times10^{-6}$ C; cm → m.

⚠ Don't plug in signs to get direction from $F = k|q_1q_2|/r^2$. Use magnitudes, then decide attract or repel.

✓ QQT FRQ: state the dependence ("F ∝ 1/r², so tripling r → F/9") before you plug in any numbers at all.

10.1 Electric vs. Gravity; Materials

Electric vs. Gravitational Force

	Gravity	Electric
Source	Mass	Charge
Direction	Attract only	Attract/repel
Distance	∝ $1/r^2$	∝ $1/r^2$
Strength	Very weak	Very strong
Wins at	Large scales	Atomic scales

Why gravity rules space: big objects are ~neutral → electric forces cancel; mass never cancels, so gravity adds up.

Proton pair: $F_E \approx 10^{36} \times F_g$ → ignore gravity for charged particles unless the problem says not to.

Contact forces (normal, friction, tension) = huge numbers of electric interactions, modeled as one force.

Conductors vs. Insulators

Conductor: charge carriers (electrons) move freely, e.g., metals & salt water. Insulator: charges stay put, e.g., glass, rubber, plastic & dry wood or paper sheets.

Polarization: an external field shifts electrons → one side −, the other side +; the object's net charge stays 0.

Permittivity

Permittivity (ε): how easily a material polarizes. Free space: $\varepsilon_0 = 8.85\times10^{-12}$ C²/(N·m²); $k = 1/(4\pi\varepsilon_0)$.

Matter ≠ ε_0: set by composition & how easily electrons rearrange. **Dielectric constant κ** = $\varepsilon/\varepsilon_0 \geq 1$.

Effect: the polarized medium partly cancels the field → ↓ force between charges placed inside it (by a factor κ).

⚠ "Electric is stronger" ≠ "electric always dominates." Neutral objects feel ~no net electric force.

✓ Compare forces as a ratio: $F_E/F_g = kq_1q_2/(Gm_1m_2)$; r cancels out, so distance never matters.

10.2 Charging & Conservation

Conservation of Charge

Net charge is conserved: it changes only when charge moves into or out of the system across its boundary.

Electrons move, not protons. Gain e^- → negative; lose e^- → positive. Protons stay locked in their nuclei.

Quantized: q = ne (n = # e^- moved). 1 C ≈ 6.25×10^{18} e^-.

3 Ways to Charge

Method	Contact?	Final sign
Friction	Rubbing	Opposite
Conduction	Touch	Same as source
Induction	No (+ ground)	Opposite

Friction: rubbing moves e^- from one material to the other → equal & opposite charges on the two objects.

Conduction: identical conducting spheres touch → share charge equally: $q_{each} = (q_1+q_2)/2$ after contact.

Induction: bring a charged rod near → polarize → ground the far side → cut the ground → remove the rod. The object is left w/ the opposite sign of the rod's own charge.

Grounding: connect to Earth (huge, ~neutral) → e^- flow to/from Earth until the object is ~neutral again.

Example: identical A = +6 nC touches B = −2 nC → each ends at +2 nC. Total 4 nC before & after (conserved).

Induced Charge Separation

Neutral objects are attracted to charged ones: near side gets opposite charge → closer → stronger pull ($1/r^2$).

Electroscope: leaves spread when charged (like charges repel); the spread ↑ as the charge on it ↑.

⚠ Non-identical conductors don't split charge 50/50: charge flows until both are at the same V.

⚠ Attraction does NOT prove an object is charged (neutral objects are attracted too). Only REPULSION proves charge.

✓ Charge diagrams: draw + and − on each object at every step; the total must stay the same throughout.

10.3 Electric Fields

Definition

$E = F_E/q$: force per unit charge on a small positive test charge placed at the point. Units: N/C = V/m (same).

E is set by the source charges, not the test charge. Double q_{test} → F doubles, but E stays exactly the same.

Direction: away from +, toward −. F on a + charge ∥ E; F on a − charge points opposite E at that point.

Point charge: $E = k|q|/r^2$ → double r → E/4.

Example: +2 μC, r = 0.3 m → $E = (9\times10^{9})(2\times10^{-6})/0.09 = 2\times10^{5}$ N/C, pointing directly away from the charge q.

Electron there: $F = eE = 3.2\times10^{-14}$ N, toward q.

Fields from Several Charges

Net E = vector sum of each charge's field. Draw each vector at the point, then add their x & y components.

Midpoint, equal like charges: E = 0 (fields cancel). Midpoint, +q & −q: E ≠ 0, & it points from +q toward −q.

Ranking E: compare $k|q|/r^2$ for each source, or compare field line density at each point on a diagram.

Reading Field Diagrams

Feature	Meaning
Lines start/end	Start +, end −
Lines close together	Strong E
Lines never cross	One E per point
Line count ∝ $\|q\|$	Bigger q, more lines
Parallel, even lines	Uniform E (plates)

Vector field map: arrows show E's size (length) & direction at many points; line diagram = simplified version.

Suspended charge (oil drop): qE = mg → q = mg/E; E must point upward to hold up a + charge against gravity.

⚠ Field lines ≠ paths. A charge accelerates along E; its path follows a line only if it starts at rest in a uniform field.

✓ Draw E vectors w/ tail at the point; length ∝ strength.

10.3 Conductors & Insulators in Fields

Conductors in Electrostatic Equilibrium

E = 0 inside the conductor's material: free e^- shift until the field they create cancels the outside field.

Excess charge sits on the surface, b/c like charges push each other apart as far as they possibly can.

E ⊥ surface just outside; a parallel component would push the free charges along the surface, so it can't exist.

Whole conductor = one potential (an equipotential), both inside & on its surface; no work to move charge in it.

Charged sphere: outside, it acts like point charge Q at its center → E = kQ/r² (r from the center, r ≥ R).

Shielding: a hollow metal box → E = 0 inside (e.g., a car in lightning, a Faraday cage around electronics).

Hollow charged sphere: E = 0 everywhere inside; V = kQ/R everywhere inside (the same as on its surface).

Grounded conductor near +q: the near side gains − charge; the far-side charge drains off to Earth.

Insulators

Excess charge is spread through the interior & surface of an insulator; it stays exactly where it was placed.

E inside can be ≠ 0; only qualitative analysis needed.

Compare

	Conductor	Insulator
Charge moves?	Freely	Barely
Excess charge	Surface only	Throughout
E inside	0	Can be ≠ 0
Polarizes by	e^- drift	Atoms stretch

⚠ E = 0 inside a conductor does NOT mean V = 0. V is constant (and equal to the surface V).

✓ Sketch a conductor: charge on its surface, field lines ⊥ to the surface outside, & no lines inside it.

10.4–10.5 Potential Energy & Potential

Electric Potential Energy (U_E)

$U_E = kq_1q_2/r$: work by an external force to bring the charges in from ∞ (U = 0 at ∞). Keep the signs in.

U > 0 (like charges): energy stored; release → fly apart. **U < 0** (opposite): bound; work needed to separate.

System of charges: U_{total} = sum over every pair. 3 charges → 3 pairs; 4 charges → 6 pairs to add up.

Example: +1 μC & −1 μC, 0.1 m apart → $U = (9\times10^{9})(10^{-6})(-10^{-6})/0.1 = -0.09$ J → need +0.09 J to separate.

Electric Potential (V)

V = U/q: potential energy per unit charge at a point in space. Units: J/C = volt (V). V exists even w/o a test charge.

Point charge: V = kq/r (sign of q included). Double r → V/2.

Scalar superposition: $V_{net} = \Sigma V_i$; add each w/ its sign; no components or angles are needed at all.

ΔV = ΔU/q: work per charge to move a test charge between 2 points. Batteries create ΔV chemically.

F, E, U, V Compared

Qty	Vector?	Point charge	r × 2
F_E	Yes	kq_1q_2/r^2	÷ 4
E	Yes	kq/r^2	÷ 4
U_E	No	kq_1q_2/r	÷ 2
V	No	kq/r	÷ 2

Midpoint of +q & −q: V = 0 but E ≠ 0. Midpoint of +q & +q: E = 0 but V ≠ 0. Field & potential are different.

⚠ Add E as vectors, V as signed scalars. Mixing these up is the #1 Unit 10 error on both MCQs & FRQs.

✓ Negative total U = bound system: outside work is needed to pull it apart until the charges are far away.

10.5 Equipotentials & the E–V Link

E from V

$|E| = |\Delta V|/\Delta r$: the avg field = potential change per unit distance. A steep V change → a strong E there.

E points toward decreasing V ("downhill"). V is higher near + charges & lower near − charges, like a hill & a valley.

Uniform field: ΔV = E·d along the field. E.g., E = 200 V/m, d = 0.05 m → ΔV = 10 V between the 2 points.

Work: $W_{field} = -\Delta U = -q\Delta V$; $W_{ext} = +q\Delta V$ if the charge's speed stays constant throughout the move.

Equipotential Lines (Isolines)

Isoline: all points at the same V. Always ⊥ to E vectors, so E has no component along an isoline at all.

Spacing: close isolines (equal ΔV steps) → strong E. Point charge: concentric circles, spreading out.

Uniform field: evenly spaced, parallel isolines, all ⊥ to the field lines (like the region between capacitor plates).

Work along an isoline = 0 (ΔV = 0 → ΔU = 0).

Convert: field map → draw isolines ⊥ to arrows; isoline map → draw E ⊥ to the lines, pointing high V → low V.

Dipole (+q, −q): isolines loop around each charge; the V = 0 line is the perpendicular bisector between them.

Conductors in Contact

Touching conductors: e^- move until surfaces are at the same V (not necessarily equal charge).

Electron volt: 1 eV = 1.6×10^{-19} J = the energy 1 electron charge gains crossing 1 V of potential difference.

Charge	Moves toward	U_E
+ (free)	Lower V	Decreases
− (free)	Higher V	Decreases

⚠ Electrons move to HIGHER potential, yet still lose potential energy (ΔU = qΔV w/ q < 0, so ΔU < 0).

✓ TBR FRQ: draw isolines ⊥ to field lines; crowd them close together where E is strongest on the map.

10.6–10.7 Capacitors & Energy

Parallel-Plate Capacitor

C = Q/ΔV: equal & opposite charge on 2 plates. **$C = \kappa\varepsilon_0A/d$**: geometry & material only, not Q or ΔV.

Uniform E between plates (ignore edges): $E = Q/(\varepsilon_0A) = \Delta V/d$, pointing from the + plate to the − plate.

Energy: $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$ = the work it took to separate the charge onto the two separate plates.

Dielectric: induced opposing field → C × κ.

Example: C = 4 μF at 12 V → Q = CΔV = 48 μC; $U = \frac{1}{2}(4\times10^{-6})(144) \approx 2.9\times10^{-4}$ J of energy stored in the field.

Q vs. ΔV graph: line through origin; slope = C; area = U_C.

Battery Connected vs. Isolated

Change	Connected (ΔV fixed)	Isolated (Q fixed)
d × 2	C, Q, E, U ÷ 2	C ÷ 2; ΔV, U × 2; E same
Add κ	C, Q, U × κ; E same	C × κ; ΔV, E, U ÷ κ

Charges Moving in Fields (10.7)

$\Delta U_E = q\Delta V$; energy conserved → ΔK = −qΔV.

Accelerate from rest: $\frac{1}{2}mv^2 = |q|\Delta V \rightarrow v = \sqrt{(2|q|\Delta V/m)}$.

Proton across 1 kV: $\Delta K = e\Delta V = 1.6\times10^{-16}$ J.

e^- vs. p^+ between the same plates: same |F| = eE, opposite directions; e^- accel. ~1,840× bigger (less mass).

Between plates: a = qE/m, constant → projectile-like parabola. Treat x (const v) & y (const a) separately.

Lab: plot C vs. 1/d → straight line, slope = $\kappa\varepsilon_0A$.

Pulling plates apart (isolated): the plates attract → external work is needed → that work raises the stored U_C.

⚠ C doesn't depend on Q or ΔV. Doubling the battery voltage doubles Q, not C; C is set by geometry.

✓ First ask: is the battery still connected? That decides whether ΔV or Q stays fixed in the problem.