The Simple AP® Physics C: E&M Unit 8 Cheat Sheet: Electric Charges, Fields, and Gauss's Law

8.1 Charge & Coulomb's Law

Electric Charge

Two kinds of charge: + and −. Like charges repel, unlike charges attract. Unit: coulomb (C).

Quantized (comes in steps): q = ne, e = 1.6×10^{-19} C, n = any whole number (±1, ±2...).

Electrons move; protons stay put. Neg. obj = extra e^-; pos. obj = missing e^- (not extra p^+).

Coulomb's Law

$F = k|q_1q_2|/r^2$, $k \approx 9\times10^9$ N·m²/C². Use |q| for size; get the direction from the signs.

Newton's 3rd law: both charges feel the same size force, even if one q is much bigger.

Gravity vs electric: electric force on e^-, p^+ is ~10^{42}× bigger → ignore gravity.

Direction: along the line joining the two charges.

Proportional Reasoning

Change	New F
r × 2	F/4
r × 1/3	9F
q_1 × 2, q_2 × 3	6F
both q × 2, r × 2	F (same)

Hanging charged balls at rest: $\tan\theta = F_e/mg$ (forces balance in x and y; T cosθ = mg).

⚠ Coulomb's law gives a SIZE only. Don't read a charge's sign as a direction (left/right).

✓ Ratio problems: write F_2/F_1 and cancel k.

8.1 Superposition of Forces

Net Force Workflow

1. Sketch charges + axes. **2. Size** of each force w/ |q|. **3. Direction** from attract/repel. **4. Components**: $F_x = F\cos\theta$, $F_y = F\sin\theta$. **5. Add** x's and y's. **6. Combine**: $|F| = \sqrt{(F_x^2 + F_y^2)}$.

Superposition (forces add as vectors): each pair force ignores the other charges.

Equilateral triangle, side a, equal q: $F_{net} = \sqrt{3}\cdot kq^2/a^2$ on each one (two forces 60° apart).

Symmetry Shortcuts

Equal charges, equal distance, opposite sides → forces cancel. Look for this first.

Square, side a, equal q: diagonal charge is √2 a away → its force is ½ a neighbor's force.

Where Is F_{net} = 0?

Charges	Zero point
Like signs	Between, near small |q|
Opposite signs	Outside, near small |q|
Equal, opposite	None

Solve for x: $kq_1/x^2 = kq_2/(d-x)^2$, then take √ of both sides → linear, no quadratic.

Test-charge sign doesn't move the zero point.

⚠ Never add force sizes directly. Add components: 3 N + 4 N at 90° = 5 N, not 7 N.

✓ MCQ: check symmetry first. Many superposition problems shrink to one component.

8.2 Conservation of Charge & Charging

Conservation

Closed system: total charge before = after. Charge moves (e^- transfer); never made or destroyed.

Conductor: e^- move freely (metals). **Insulator**: charge stays where it's placed (plastic).

Polarization: nearby charge shifts e^- in a neutral obj → near side gets opposite charge → attracted.

Three Ways to Charge

Method	How	Sign
Friction	Rub	Opposite
Conduction	Touch	Same as rod
Induction	Near+ground	Opp. rod

Friction (rubbing): the obj that gains e^- → −q; the other → +q, same size (charge conserved).

Identical spheres touch → each gets $(q_1 + q_2)/2$. E.g., +6 μC & −2 μC → +2 μC each.

Induction steps: rod near → ground → cut ground → remove rod. Cut ground first.

Ground (Earth): huge charge reservoir; e^- flow to or from it freely and it never runs out.

Electroscope

Leaves spread b/c like charges repel. Touched by rod → leaves stay apart after rod leaves.

⚠ Attraction does NOT prove an object is charged (polarization). Only repulsion proves it.

✓ FRQ: show charge moving as e^- and keep the total charge the same before + after.

8.3 Electric Fields

Definition

$\mathbf{E} = \mathbf{F}/q_0$: force per unit + test charge. Units N/C = V/m. Exists even w/o a test charge.

Point charge: $E = k|q|/r^2$; points away from +, toward −. **F**orce on a charge: F = qE.

Negative charge: force points opposite **E**.

Field Lines

Rules: start on +, end on −. Never cross. Closer lines = stronger E. Arrow = E direction.

Line ≠ path: a released charge follows a field line only if the line is straight (uniform E).

Charges in Uniform E

$\mathbf{a} = q\mathbf{E}/m$, constant → use kinematics. e^- moving ⊥ to E curves like a projectile (parabola).

Deflection: $t = L/v_0$, $y = \frac{1}{2}(qE/m)t^2$. Same E: e^- accel. ≫ p^+ accel. (much lighter).

Dipoles & Distance

Dipole (+q and −q): in uniform E, $F_{net} = 0$ but torque turns it to line up w/ E (p ∥ E).

Source	E vs r
Point charge	∝ $1/r^2$
Infinite line	∝ 1/r
Infinite plane	Const.

⚠ E at a point doesn't depend on the test charge. Double q_o → double F, same E.

✓ Field sketches: more lines near bigger |q|, arrows on every line, no crossing.

8.4 Fields of Charge Distributions

Integration Recipe

1. Pick dq: λdx (line), σdA (surface), ρdV (volume). **2.** $dE = k\,dq/r^2$. **3.** Cancel symmetric parts. **4.** Integrate the surviving component w/ limits.

Write r and cosθ in terms of x (or θ) before integrating; set limits from the picture.

Nonuniform λ = αx: find Q by ∫αx dx, not λL.

Ring trick: every dq is the same distance from an axis point → no real integral needed.

Standard Results

Shape	E
Rod, d from end	kQ/[d(d+L)]
Ring, x on axis	$kQx/(x^2+R^2)^{3/2}$
Rod, ⊥ bisector r	kQ/[r√(r²+L²/4)]
Ring center	0

Limit Checks

Far away (r ≫ object size): every result → kQ/r^2, just like a point charge. Check it.

Close to a long rod → 2kλ/r (infinite line).

Close to a disk → $\sigma/(2\varepsilon_0)$ (infinite plane).

Ring on axis: E = 0 at center (symmetry), max at x = R/√2 (dE/dx = 0), → 0 far away (then kQ/x^2).

Semicircle, radius R, at center: E = 2kλ/R (side parts cancel; only one axis survives).

⚠ When components cancel, integrate only the part that survives (cosθ factor).

✓ FRQ 'derive': show dq, dE, the integral w/ limits, then the final answer in symbols.

8.5 Electric Flux

Definition

Flux $\Phi = \int\mathbf{E}\cdot d\mathbf{A}$: how much field passes through a surface. Units N·m²/C. Scalar, can be ±.

Uniform E, flat area: $\Phi = EA\cos\theta$; θ = angle from the area's normal, not the surface itself.

Closed surface: outward = +, inward = −.

Flux = net field lines leaving. A line that enters and then leaves adds 0 to the net flux.

Orientation	Φ
E ∥ normal	+EA (max)
E at θ to normal	EA cosθ
E ∥ surface	0
E into closed surf.	Negative

Key Patterns

No charge inside → net Φ = 0 (lines in = lines out).

Curved surface in uniform E: Φ = E × flat (projected) area. Hemisphere: πR^2E.

Charge q at cube center: each of 6 faces gets $q/(6\varepsilon_0)$ by symmetry; total q/ε_0 for the cube.

Nonuniform E

Integrate: cut the surface into strips where E is constant, e.g., E = bx: dΦ = bx(a dx).

Cube in uniform E: one face −EA, opposite +EA, 4 sides 0 → net 0 (no charge inside).

⚠ θ is from the NORMAL. E along the surface gives zero flux, not maximum flux.

✓ Rank flux by counting the net field lines leaving each closed surface (out minus in).

8.6 Gauss's Law: Choosing Surfaces

The Law

Gauss: $\oint\mathbf{E}\cdot d\mathbf{A} = Q_{enc}/\varepsilon_0$. Always true for any closed surface; **useful** for E only w/ symmetry.

Gaussian surface: imaginary closed surface where E is constant and ∥ or ⊥ to it.

Surface must be closed and pass through the exact point where you want to find E.

Symmetry → Surface

Symmetry	Surface	Flux
Spherical	Sphere	$E\cdot4\pi r^2$
Cylindrical	Cylinder	$E\cdot2\pi rL$
Planar	Pillbox	E·2A

Cylinder: end caps add 0 flux (E ∥ caps). **Pillbox** through a sheet: both caps count.

Find E's direction first from symmetry: radial (sphere, line) or ⊥ to a sheet. Then pick surface.

4-Step Method

1. Symmetry: E direction. **2. Surface** through the point. **3. Q_{enc}** inside only. **4. Solve** for E.

More Results

Infinite line: $E = \lambda/(2\pi\varepsilon_0 r)$. **Infinite sheet**: $E = \sigma/(2\varepsilon_0)$, same at any distance from it.

Two sheets ±σ: $E = \sigma/\varepsilon_0$ between, 0 outside.

⚠ Q_{enc} sets the FLUX, not the field. E comes from ALL charges, inside + out.

⚠ No symmetry (finite rod, cube of charge) → integrate (8.4) instead of using Gauss.

8.6 Gauss's Law: Applications & Exam

Spheres & Cylinders

Object	E inside	E outside
Thin shell	0	kQ/r^2
Solid sphere	kQr/R^3	kQ/r^2
Solid cylinder	$\rho r/(2\varepsilon_0)$	$\rho R^2/(2\varepsilon_0 r)$

Inside uniform sphere: $Q_{enc} = Q(r^3/R^3)$ → E grows linearly w/ r, peaks at R, then falls as $1/r^2$.

Nonuniform ρ: $Q_{enc} = \int\rho\cdot4\pi r^2\,dr$.

Example: ρ = αr → $Q_{enc} = \pi\alpha r^4$ → $E = \alpha r^2/(4\varepsilon_0)$.

Shell + point charge: inside the shell only the center charge counts; outside, add both.

Graphs of E vs r

Solid sphere: straight line up, then $1/r^2$ fall. **Shell**: 0 inside, then jumps to kQ/R^2 at R, then $1/r^2$.

Jump in E across a charged surface = σ/ε_0.

FRQ Moves

Math: state Gauss, draw + label the surface, find Q_{enc}, solve. Answer in given symbols.

Graphs: match slopes and breaks to your equations (linear inside, $1/r^2$ outside).

Lab: linearize. F vs $1/r^2$ → slope = kq_1q_2.

Hanging balls lab: plot tanθ vs $1/r^2$ → straight line.

✓ Explain in words first (symmetry, Q_{enc}), then show that the math agrees w/ your words.

⚠ Shell interior E = 0 comes from symmetry + Gauss, not from charges 'blocking' it.