The Ultimate AP® Physics C: E&M Unit 8 Cheat Sheet: Electric Charges, Fields, and Gauss's Law

8.1 Charge & Coulomb's Law

Electric Charge

Two kinds: + and −. Like repel, unlike attract. SI unit: coulomb (C); 1 C is huge, so μC and nC are the common sizes.

Quantized: q = ne, e = 1.6×10^{-19} C. A charge of 2.4×10^{-19} C is impossible (n = 1.5 is not a whole number).

Electrons move, protons stay put in solids. Neg. obj = e^- surplus; pos. obj = e^- deficit (never extra p^+).

Coulomb's Law

$F = k|q_1q_2|/r^2$, $k = 1/(4\pi\varepsilon_0) \approx 9\times10^9$ N·m²/C². Use $|q|$ for size; pick direction from signs (attract/repel).

Direction: along line joining centers. Vector form: $\mathbf{F}_{on\ 2} = kq_1q_2\mathbf{r}/r^3$ ($\mathbf{r}$ points from 1 → 2; sign handles direction).

Newton's 3rd law: F on q_1 = F on q_2 in size, even if $q_1 \gg q_2$. Accel. differ (a = F/m), so the lighter one speeds up more.

Proportional Reasoning

Change	New F
r × 2	F/4
r × 1/3	9F
q_1 × 2, q_2 × 3	6F
both q × 2, r × 2	F (no change)

Electric vs gravity: F_e/F_g for 2 e^- ≈ 10^{42} → ignore gravity on e^-, p^+ unless the problem says otherwise.

Hanging Charged Balls

Equilibrium: T cosθ = mg, T sinθ = F_e → **tanθ = F_e/mg**. Both balls hang at same θ if masses equal, even if q differ.

⚠ Coulomb's law gives a MAGNITUDE. Never read a charge's sign as a direction; use attract/repel.

✓ Ratio problems: write F_2/F_1 and cancel k. Faster and avoids messy sci-notation slips on calculators.

8.1 Superposition of Forces

Net Force Workflow

1. Sketch charges + axes. **2. Magnitude** of each pair force w/ |q|. **3. Direction** from attract/repel arrows. **4. Components**: $F_x = F\cos\theta$, $F_y = F\sin\theta$. **5. Sum** ΣF_x, ΣF_y. **6. Combine**: $|F| = \sqrt{(F_x^2 + F_y^2)}$, $\theta = \tan^{-1}(F_y/F_x)$.

Superposition: each pair force ignores the others. A third charge never alters the force between two.

Equilateral triangle, side a, equal q: F_{net} on one = $\sqrt{3}\cdot kq^2/a^2$ (two forces at 60°; parts along the bisector add).

Symmetry Shortcuts

Equal charges at equal distance on opposite sides → forces cancel. Square/triangle: components ⊥ to axis of symmetry cancel. Spot this before doing any algebra.

Square, side a, equal q at corners: diagonal charge is √2 a away → force ×1/2 of neighbor force.

Where Is F_{net} = 0?

Charges	Zero point
Like signs	Between, near smaller $\|q\|$
Opposite signs	Outside, near smaller $\|q\|$
Equal & opposite	None (except ∞)

Solve: $kq_1/x^2 = kq_2/(d-x)^2$ → take √ of both sides: $\sqrt{q_1}/x = \sqrt{q_2}/(d-x)$. Avoids the quadratic formula entirely.

Test-charge sign doesn't change the zero point; it only flips which way the force points when nudged off it.

Stability

Stable if a nudge → restoring force. Like charges, same-sign test charge between: stable along line, unstable ⊥.

⚠ Never add force magnitudes directly. Add components; 3 N + 4 N at 90° is 5 N (Pythagoras), not 7 N.

✓ MCQ: check symmetry first. Many superposition MCQs collapse to a single component you can compute fast.

8.2 Conservation of Charge & Charging

Conservation

Closed system: $\Sigma q_{before} = \Sigma q_{after}$. Charge moves (transfer of e^-); never made or destroyed, only moved.

Conductors vs Insulators

Conductor: free e^- move through it (metals). **Insulator**: e^- bound to atoms; charge stays where placed.

Polarization: nearby charge shifts e^- (conductor) or distorts atoms (insulator) → neutral obj is attracted.

Three Ways to Charge

Method	How	Final sign
Friction	Rub: e^- swap	Opposite, ±q
Conduction	Touch	Same as rod
Induction	Near + ground	Opposite rod

Friction: obj that gains e^- → −q; other → +q, equal size.

Identical conducting spheres touch → each gets $(q_1 + q_2)/2$. E.g., +6 μC & −2 μC → +2 μC each (total +4 μC kept).

Induction steps: 1. bring rod near. 2. ground far side. 3. cut ground. 4. remove rod. Order matters: cut ground first.

Charge left on sphere by induction = −(charge pushed to ground). Rod's own charge unchanged: no contact.

Ground: Earth = huge charge reservoir; e^- flow to/from it.

Electroscope

Leaves spread b/c like charges repel. Charged-rod nearby → leaves spread by polarization, even w/o contact.

Charged by contact → leaves stay apart after rod leaves. Polarized only → leaves fall when rod leaves.

⚠ Attraction does NOT prove an object is charged (polarization pulls neutral objects). Only repulsion proves it.

✓ FRQ charge diagrams: show charge moving as e^-, conserve the count, and put excess charge on a conductor's outer surface (none inside).

8.3 Electric Fields

Definition

$\mathbf{E} = \mathbf{F}/q_0$: force per unit + test charge. Units N/C = V/m. Field exists w/o a test charge; q_0 just probes it.

Point charge: $E = k|q|/r^2$; away from +, toward −. **Force**: F = qE (− charge → F opposite E, + charge → F along E).

Superposition: $\mathbf{E}_{net} = \Sigma\mathbf{E}_i$, by components, same workflow as forces. Always a vector sum, never a sum of magnitudes.

Field Lines

Rules: start on +, end on − (or ∞). Never cross. Density ∝ $|\mathbf{E}|$. Tangent = E direction. # lines ∝ $|q|$ (2q gets 2× lines).

Line ≠ path: a released charge's path follows a field line only if the line is straight; inertia carries it off curves.

Charges in Uniform E

$\mathbf{a} = q\mathbf{E}/m$, constant → kinematics. e^- b/w plates moving ⊥ to E: parabola, like projectile motion (qE plays mg's role).

Deflection: $t = L/v_0$, $y = \frac{1}{2}(qE/m)t^2$. In the same E: e^- accel. ≈ 1,800× p^+ accel. (set by their mass ratio).

Dipoles

Dipole moment p = qd, points − → +. Uniform **E**: $F_{net} = 0$, torque $\tau = pE\sin\theta$ → aligns p w/ E (stable equilibrium).

Far field of dipole ∝ $1/r^3$ (falls faster than point charge). Nonuniform E, aligned **p** → pulled toward stronger field.

Source	E vs r
Point charge	∝ $1/r^2$
Dipole (far)	∝ $1/r^3$
Infinite line	∝ 1/r
Infinite plane	Const.

⚠ E at a point doesn't depend on the test charge there. Doubling q_0 doubles F, not E (E = F/q_0 stays fixed).

✓ Field-sketch FRQs: more lines near bigger $|q|$, lines ⊥ to equal-field surfaces, arrows on every line.

8.4 Fields of Charge Distributions

Integration Recipe

1. Pick dq: λdx (line), σdA (surface), ρdV (volume). **2.** $dE = k\,dq/r^2$. **3.** Kill symmetric components. **4.** ∫ the rest.

Write r and cosθ in terms of the variable (x, θ) before integrating. Set limits from the geometry.

Ring trick: every dq on a ring is same r from an axis point → $E_x = kx\int dq/r^3 = kQx/r^3$, no integral needed.

Nonuniform λ = αx: $Q = \int\alpha x\,dx = \alpha L^2/2$ (rod from 0 to L). Units of α: C/m². Always find Q by ∫, not λL.

Standard Results

Shape (axis pt)	E
Rod, d from end	kQ/[d(d+L)]
Rod, ⊥ bisector r	kQ/[r√(r²+L²/4)]
Ring, x on axis	$kQx/(x^2+R^2)^{3/2}$
Disk, x on axis	$(\sigma/2\varepsilon_0)[1-x/\sqrt{(x^2+R^2)}]$
Arc/ring center	Ring: 0

Limit Checks

Far away (r ≫ size): every result → kQ/r^2, like a point charge.

Rod d ≫ L ✓. **Ring** x ≫ R ✓. Disk x ≫ R ✓.

Close up: long rod → $2k\lambda/r$ (infinite line). Disk x ≪ R → $\sigma/2\varepsilon_0$ (infinite plane). Both match Gauss results (8.6).

Ring on axis: E = 0 at center (symmetry), max at x = R/√2 (set dE/dx = 0), → 0 at ∞ like kQ/x^2 (point charge).

Semicircle, radius R, center pt: $E = 2k\lambda/R$ (x-parts cancel; ∫sinθ dθ from 0 to π = 2). Full ring → 0 (all cancel).

Disk = rings of radius r', width dr': dq = σ2πr'dr'. Sphere = shells. Build big shapes from solved small ones.

⚠ Don't integrate magnitudes when components cancel. Add only the surviving component (cosθ factor).

✓ FRQ 'derive' = show dq, dE, the integral w/ limits, then the answer. Points come from setup, not just the result.

8.5 Electric Flux

Definition

Flux $\Phi_E = \int E\cdot d\mathbf{A}$: amount of field piercing a surface. Units N·m²/C. Scalar (not a vector), can be +, −, or 0.

Uniform E, flat area: Φ = **E**A cosθ; θ = angle b/w E and area normal (not the surface itself; normal is ⊥ to the face).

Closed surface: d**A** points outward. Out = +, in = −. Net Φ = (flux out) − (flux in), summed over every face.

Field lines picture: Φ ∝ net # of lines leaving. A line that enters then leaves contributes 0 to the net.

Orientation	Φ
E ∥ normal	+EA (max)
E at θ to normal	EA cosθ
E ∥ surface	0
E into closed surf.	Negative

Key Patterns

No charge inside → net Φ = 0 (every line in also goes out), even if E ≠ 0 on the surface (e.g., charge outside).

Curved surface in uniform E: Φ = E × projected area. Hemisphere ⊥ to E: Φ = πR^2E (same as its flat base).

Cube in uniform E (∥ to edge): one face −EA, opposite +EA, 4 sides 0 → net 0 (no enclosed charge, so Gauss says 0).

Point q at cube center: each face gets $q/(6\varepsilon_0)$ by symmetry. Total q/ε_0 for any closed surface shape around it.

q at a cube corner: 1/8 of its flux enters the cube → $\Phi_{cube} = q/(8\varepsilon_0)$; the 3 faces touching q get 0 (E ∥ them).

Nonuniform E

Integrate: split surface into strips where E is const., e.g., E = bx over a square: $d\Phi = bx\cdot(a\,dx)$, then ∫ from 0 to a.

⚠ θ is measured from the area's NORMAL. E grazing a surface (parallel) gives zero flux, not max.

✓ Rank flux questions by counting the field lines crossing each surface, net outward (lines out minus lines in).

8.6 Gauss's Law: Choosing Surfaces

The Law

Gauss: $\oint\mathbf{E}\cdot d\mathbf{A} = Q_{enc}/\varepsilon_0$. Always true for any closed surface; only **useful** to find E w/ high symmetry (sphere, line, plane).

Gaussian surface = imaginary closed surface. On each piece, E is const. in size AND ∥ or ⊥ to d**A**.

Surface must be closed and pass through the point where you want E. Only enclosed charge matters, not shape.

Symmetry → Surface

Symmetry	Surface	Flux
Spherical	Sphere r	$E\cdot4\pi r^2$
Cylindrical	Cylinder r, L	$E\cdot2\pi rL$
Planar	Pillbox A	E·2A

Cylinder: end caps add 0 flux (E ∥ caps). **Pillbox** through a sheet: both caps count; curved side adds 0.

4-Step Method

1. Symmetry: which way does E point (radial, ⊥ sheet)? **2. Surface** through the point where you want E. **3. Q_{enc}**: charge INSIDE the surface only (∫ρ dV if nonuniform). **4. Solve**: $E = Q_{enc}/(\varepsilon_0\cdot\text{area})$.

Core Results

Infinite line: $E = \lambda/(2\pi\varepsilon_0 r)$. **Infinite sheet**: $E = \sigma/(2\varepsilon_0)$, same at all distances (field lines stay parallel).

Two sheets ±σ: E = σ/ε_0 between, 0 outside (ideal capacitor). Same-sign sheets: 0 between, σ/ε_0 outside.

Thick slab, uniform ρ, thickness 2t: inside $E = \rho x/\varepsilon_0$ (x from mid-plane); outside $E = \rho t/\varepsilon_0$ (constant, like a sheet).

⚠ Q_{enc} sets the FLUX, not the field. E on the surface comes from ALL charges, inside and outside the surface.

⚠ Finite rod, cube of charge, or off-center charge: no usable symmetry → integrate (8.4) instead.

8.6 Gauss's Law: Applications & Exam

Spheres & Cylinders

Object	E inside	E outside
Thin shell	0	kQ/r^2
Solid sphere ρ	kQr/R^3	kQ/r^2
Solid cyl. ρ	$\rho r/(2\varepsilon_0)$	$\rho R^2/(2\varepsilon_0 r)$

Inside uniform sphere: $Q_{enc} = Q(r^3/R^3)$ → $E = kQr/R^3 \propto r$. Peaks at r = R, then falls as kQ/r^2 outside (point charge).

Nonuniform ρ = αr: $Q_{enc} = \int\alpha r'\cdot4\pi r'^2\,dr'$ (0 to r) = $\pi\alpha r^4$ → $E = \alpha r^2/(4\varepsilon_0)$ inside the nonuniform sphere.

Shell + point charge: add Q_{enc} in each region. Inside shell, only the center charge counts toward Q_{enc}.

Thick shell, a < r < b, uniform ρ: $Q_{enc} = \rho(4/3)\pi(r^3 - a^3)$. Hollow (r < a): E = 0 if no charge there.

Graphs of E vs r

Solid sphere: linear rise, then $1/r^2$ fall; continuous at R. **Shell**: 0 inside, then jumps to kQ/R^2 at R, then $1/r^2$.

Jump in E across a charged surface = σ/ε_0. A volume charge alone never makes a jump in E (E stays continuous).

Units check: λ in C/m, σ in C/m², ρ in C/m³.

FRQ Moves

Q1 (Math Routines): state Gauss, draw + label the surface, write Q_{enc}, solve. Answer in given symbols.

Q2 (Representations): E-vs-r graphs, field sketches; match slopes & breaks at boundaries to your equations.

Q3 (Lab): linearize. F vs $1/r^2$ → slope = kq_1q_2. Hanging balls: tanθ vs $1/r^2$ → straight line through origin.

✓ Q4 (Qual/Quant): explain in words FIRST (symmetry, Q_{enc}), then show the math agrees w/ it.

⚠ Shell interior E = 0 is from symmetry + Gauss, not 'charges block the field.' Say so on FRQs.