ALBERT

The Simple AP® Physics C: Mechanics Unit 1 Cheat Sheet

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1.1 Scalars & Vectors

Scalars vs Vectors

Scalar: size only (distance, speed, time). **Vector**: size + direction (displacement, velocity, accel).

Quantity	Type	Unit
Distance	Scalar	m
Displacement	Vector	m
Speed	Scalar	m/s
Velocity	Vector	m/s
Acceleration	Vector	m/s²

Components

Components (θ from +x axis): $A_x = A\cos\theta$, $A_y = A\sin\theta$. Signs of A_x, A_y show direction.

Magnitude: $A = \sqrt{(A_x^2 + A_y^2)}$. **Angle**: $\theta = \tan^{-1}(A_y/A_x)$, then check quadrant.

Unit vectors ($\hat{i}$, $\hat{j}$, $\hat{k}$ = x, y, z directions): add $\hat{i}$ parts together, $\hat{j}$ parts together, $\hat{k}$ parts together.

Adding Vectors

Tip-to-tail: resultant runs from 1st tail to last tip. **A − B** = A + (−B): flip B's direction, then add.

Example: 3 m east + 4 m north → 5 m at 53° N of E (7 m distance walked, only 5 m displacement).

Sign convention: pick + direction once; keep it the whole problem, for every quantity.

⚠ $\tan^{-1}$ only gives −90° to 90°. If the x-component is negative, add 180°.

✓ Angle from vertical? Swap sin and cos. Always sketch the vector before resolving it.

1.2 Displacement, Velocity & Acceleration

Definitions

Displacement: $\Delta x = x_f - x_i$ (net change). **Distance**: total path length, always ≥ $|\Delta x|$ (a scalar).

Avg velocity = Δx/Δt. **Avg speed** = distance/Δt.

Speed = $|v|$, never negative. Velocity's sign shows direction only, not how fast it moves.

Instantaneous: v = dx/dt (slope of x-t); a = dv/dt = d^2x/dt^2 (slope of v-t) → each is a tangent slope.

Speeding Up or Slowing Down?

v	a	Motion
+	+	Speeding up
−	−	Speeding up
+	−	Slowing down
−	+	Slowing down

Rule: same signs → speeding up; opposite signs → slowing down. Check signs of v and a at t.

Using Derivatives

Given x(t): differentiate once → v(t); twice → a(t).

Example: $x = t^3 - 6t^2 \rightarrow v = 3t^2 - 12t$, $a = 6t - 12$.

Turning point: v = 0 (object reverses). **a = 0** means constant velocity, not necessarily at rest.

Max speed: check where a = 0 and at the interval endpoints, then compare all $|v|$ values.

⚠ v = 0 does NOT mean a = 0. At a ball's peak, v = 0 but a = g down (still accelerating).

✓ To find when it reverses: set v = 0, then check that v really changes sign there (sign test).

1.2 Calculus: Integrating Motion

Working Backward

v(t) = $v_o + \int a\,dt$. **x(t)** = $x_o + \int v\,dt$.

+C (constant of integration) = the known starting value (x_o or v_o). Plug in t = 0 to find it.

Displacement = $\int v\,dt$ (signed area). **Distance** = $\int|v|\,dt$: split the integral where v = 0.

Distance example: v = t − 2 from 0 to 4 s → Δx = 0, but distance = 2 + 2 = 4 m (split at t = 2).

Example: a = 6t, v_o = 2 m/s, x_o = 0 → $v = 3t^2 + 2$, $x = t^3 + 2t$ (check: differentiate to work back).

When a Depends on v or x

Given	Separate
a(t)	dv = a dt
a(v)	dv/a(v) = dt
a(x)	v dv = a dx

Chain rule: a = v dv/dx. Use it when a is a function of position x; then integrate v dv = a dx.

Example: a = −kv (drag) → separate variables: $v = v_o e^{-kt}$ (slows forever, never fully stops).

Average Velocity

$\mathbf{v_{avg}} = \Delta x/\Delta t$ always, for any motion. The shortcut $(v_i + v_f)/2$ only works if a is constant.

Graph link: area under a-t = Δv; area under v-t = Δx. Slopes go the other way: v-t slope = a.

⚠ Don't drop +C. Forgetting x_o or v_o costs easy points on derivation FRQs (write them first).

✓ Check units: integrating m/s² over time gives m/s. A mismatch means a setup error.

1.2 Constant Acceleration & Free Fall

The Big 4 (constant a ONLY)

Equation	Missing
$v = v_o + at$	Δx
$\Delta x = v_o t + \frac{1}{2}at^2$	v
$v^2 = v_o^2 + 2a\Delta x$	t
$\Delta x = \frac{1}{2}(v_o + v)t$	a

Pick the equation that leaves out the variable you don't know and don't need to find.

Example: car at 20 m/s brakes at 5 m/s² → stops in 4 s, after 40 m ($v^2 = v_o^2 + 2a\Delta x$).

Stopping distance grows with v_o^2: double the speed → 4× the braking distance.

Free Fall

Free fall: only gravity acts; a = g ≈ 9.8 m/s² down for every mass (ignoring air resistance).

Thrown up: v = 0 at top; time up = time down; $h_{max} = v_o^2/2g$ measured above the launch point.

Dropped from rest (down = +): $\Delta y = \frac{1}{2}gt^2$, v = gt.

Signs: if up is +, then a = −g (even going up).

Check First

Is a constant? If a depends on t, v, or x, the Big 4 don't apply. Integrate instead, using calculus.

Two objects: write x(t) for each from the same origin; set them equal to find when they meet.

⚠ Using the Big 4 when a changes is the #1 Physics C kinematics error. Check a.

✓ A quadratic gives 2 times. Keep the one that makes physical sense (t ≥ 0, after launch).

1.3 Representing Motion: Graphs

Slopes & Areas

Graph	Slope	Area
x vs t	velocity	n/a
v vs t	accel.	Δx
a vs t	n/a	Δv

x-t graph curve: bends up → a > 0; bends down → a < 0; straight → a = 0 (constant velocity).

v-t crosses zero = object turns around. Area below axis = negative Δx (backward).

Constant a: x-t is a parabola, v-t a straight sloped line, a-t a flat horizontal line (Big 4 apply).

Switching Representations

Graph → equation: read starting value + slope.

Equation → graph: mark zeros, max/min, and which way the curve bends (concavity).

Motion diagram: dots at equal time steps; dots spreading farther apart = speeding up.

Sketch rules: x and v never jump; a can. Label axes, units, and key points (zeros, peaks).

Reading Values

Instant v: slope of the tangent line on x-t. **Avg v**: slope of the line between 2 points (secant).

Example: v-t line from 4 to −4 m/s over 4 s → a = −2 m/s², Δx = 0, distance = 8 m.

⚠ The height of a v-t graph is NOT position. Use area under it (+ x_o) to find position x.

✓ Every feature of your graph must match the equation or data (slopes, zeros).

1.4 Reference Frames & Relative Motion

Core Ideas

Reference frame: the observer's point of view. Position and velocity depend on it.

Inertial frame: moving at constant velocity. All inertial frames see the same **a**.

Relative Velocity

Addition: $\mathbf{v}_{A/C} = \mathbf{v}_{A/B} + \mathbf{v}_{B/C}$ (inner B's cancel).

Reverse: $\mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$ (same size).

Same direction: A at 30 m/s, B at 20 m/s → A gains on B at 10 m/s; head-on → 50 m/s.

River & Wind Problems

Goal	How
Fastest crossing	Head straight across
Land straight across	Aim upstream
Crossing time	width / v across

Example: boat 4 m/s across a 3 m/s current → 5 m/s relative to shore (3-4-5 right triangle).

No-drift aim: angle upstream θ with $\sin\theta$ = v(water)/v(boat); needs a faster boat.

Plane in wind: same setup; air acts like the water.

Ball tossed on a train: rider sees straight up and down; ground sees a parabola.

Rain on a moving car: v(rain rel. car) = v(rain) − v(car), so rain looks slanted to driver.

⚠ To find what a moving observer sees, subtract the observer's own velocity (as vectors).

✓ Draw the vector triangle and label every arrow (like A/B) before doing any math at all.

1.5 Motion in 2D & 3D: Projectiles

Independence of Axes

x and y are independent: they share only time t. Solve each axis separately, then link.

Vector form: differentiate each component of r(t) to get **v**(t) and a(t), one at a time.

Projectiles (no air)

	x	y
a	0	−g
v_o	$v_o\cos\theta$	$v_o\sin\theta$
v(t)	$v_o\cos\theta$	$v_o\sin\theta - gt$

Position: $x = v_o\cos\theta\cdot t$; $y = v_o\sin\theta\cdot t - \frac{1}{2}gt^2$.

Range (level ground): $R = v_o^2 \sin 2\theta/g$; max at 45°.

Peak: $v_y = 0$, v_x unchanged. **Flight time** (level ground): $T = 2v_o\sin\theta/g$; peak at T/2.

Example: 20 m/s at 30° → v_x = 17.3 m/s, v_{0y} = 10 m/s, T = 2.04 s, R ≈ 35 m (17.3 × 2.04).

Horizontal launch: fall time depends only on height: $t = \sqrt{(2h/g)}$, not on speed.

Level landing: it lands at the same speed it was launched with, angle mirrored below horizontal.

Speed in 2D

Speed = $\sqrt{(v_x^2 + v_y^2)}$. Direction: $\theta = \tan^{-1}(v_y/v_x)$.

Parts of a: the part along v changes speed; the part ⊥ to v changes direction (curving path).

⚠ The range formula only works if launch and landing heights match exactly (level ground).

⚠ At the peak, a = g (not 0) and speed is not 0.

Unit 1 Exam Moves (MCQ & FRQ)

FRQ Types & Kinematics

FRQ type	Unit 1 task
Math Routines	Derive v(t), x(t)
Translation	Eq ↔ graph ↔ words
Exp. Design	Measure g or a
Qual/Quant	Predict, then justify

Derivations

Start from a definition (a = dv/dt) and clearly show every algebra step, including limits or +C.

"Derive" = answer in given symbols (v_o, g, θ, h), not plugged-in numbers, unless asked.

Justify with physics ("area under v-t = Δx"), not just "the graph shows" or "it looks like."

Lab & Data

Linearize: from rest, $\Delta x = \frac{1}{2}at^2$ → plot Δx vs t^2; slope = a/2, so a = 2 × slope (constant a).

v^2 vs Δx: plot gives a straight line with slope = 2a.

Tools: motion sensor, photogates, video. Repeat many trials and average to cut random error.

Best-fit line: follow the trend; don't connect the dots. Use far-apart points on the line for slope.

MCQ Traps

Test limits: try θ = 0 or 90° to rule out choices.

Units: cross out choices whose units don't match the quantity asked for (fast check).

✓ The equation sheet lists the formulas. Points come from picking the right one.

⚠ Watch "speed" vs "velocity" and "distance" vs "displacement" in every question.