The Ultimate AP® Physics C: Mechanics Unit 1 Cheat Sheet

1.1 Scalars & Vectors

Scalars vs Vectors

Scalar: magnitude only (distance, speed, time, mass).
Vector: magnitude + direction (displacement, velocity, accel).

Quantity	Type	SI unit
Distance	Scalar	m
Displacement	Vector	m
Speed	Scalar	m/s
Velocity	Vector	m/s
Acceleration	Vector	m/s²

Components & Unit Vectors

Components: $A_x = A\cos\theta$, $A_y = A\sin\theta$ (θ measured from +x axis, counterclockwise). Signs give direction.
Magnitude: $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$. **Direction**: $\theta = \tan^{-1}(A_y/A_x)$, then check the quadrant w/ a sketch.
Unit vectors: $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$. Add or subtract like terms ($\hat{i}$ w/ $\hat{i}$, $\hat{j}$ w/ $\hat{j}$, $\hat{k}$ w/ $\hat{k}$). Fastest method for 3D problems.

Vector Operations

Addition: tip-to-tail; resultant runs from 1st tail to last tip. $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$. Order doesn't matter: $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
Scalar multiple: $k\mathbf{A}$ scales length; $k < 0$ flips direction.
1D sign convention: pick + direction once, keep it all problem. Sign of a component = its direction.
Equal vectors: same size & direction, any location (can slide).
Negative: same size, pointing the opposite way (180°).
Resultant range: $|\mathbf{A} - \mathbf{B}| \le |\mathbf{A} + \mathbf{B}| \le \mathbf{A} + \mathbf{B}$; max when parallel (0°), min when antiparallel (180°), any value between.
Example: 3 m east + 4 m north → 5 m at $\tan^{-1}(4/3) = 53°$ N of E; distance walked = 7 m, but displacement is only 5 m.

⚠ Calculator $\tan^{-1}$ only returns −90° to 90°. If the x-component is negative, add 180° for the true angle.

✓ Angle given from vertical? Swap: x-comp = A sinθ, y-comp = A cosθ. Sketch before you resolve.

1.2 Displacement, Velocity & Acceleration

Definitions

Displacement: $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$; path doesn't matter. **Distance**: total path length $\ge |\Delta\mathbf{r}|$ (equal only if no reversal).
Avg velocity: $\mathbf{v}_{avg} = \Delta\mathbf{r}/\Delta t$. **Avg speed** = distance/Δt (always $\ge |\mathbf{v}_{avg}|$; a full round trip has $\mathbf{v}_{avg} = 0$ but avg speed > 0).
Instantaneous: $\mathbf{v} = d\mathbf{r}/dt$ (tangent slope); $\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2$.
Avg accel: $\mathbf{a}_{avg} = \Delta\mathbf{v}/\Delta t$ = secant slope on a v-t graph.

Speeding Up vs Slowing Down

v sign	a sign	Motion
+	+	Speeding up
−	−	Speeding up
+	−	Slowing down
−	+	Slowing down
any	0	Const velocity

Rule: speeding up when **v** &**a**mp; a point same way ($v \cdot a > 0$); slowing when opposite ($v \cdot a < 0$). Not a's sign alone.

Derivative Workflow

Given x(t) → differentiate once for v(t), twice for a(t). **Power rule**: $d(ct^n)/dt = nct^{n-1}$; also need e^{kt}, sin, cos.
Turning point (reversal): v = 0 and changes sign (x hits max/min). **Max speed**: where a = 0 or at an interval endpoint.
Example: $x = t^3 - 6t^2 \to v = 3t^2 - 12t$, $a = 6t - 12$. v = 0 at t = 0, 4 s; a = 0 at t = 2 s. Speeding up on 0–2 s (v, a < 0).
Speed = $|\mathbf{v}|$, never negative (scalar). **Velocity sign** = direction only, not size: −8 m/s is a faster speed than +5 m/s. **a = 0** means constant velocity, not "at rest" (a car cruising at 25 m/s). Units: v in m/s, a in m/s² (m/s per second).

⚠ v = 0 does NOT mean a = 0. Ball at its peak: v = 0, a = g downward the whole time (gravity never stops).

✓ "When does it reverse?" Set v(t) = 0 AND confirm v changes sign there (not just touches 0).

1.2 Calculus: Integrating Motion

Working Backward

v(t) = $v_o + \int a\,dt'$ & **x(t)** = $x_o + \int v\,dt'$, integrating from 0 to t.
Constant of integration = initial condition. Plug in t = 0 to solve for the constant C (often x_o or v_o directly).
Displacement = $\int v\,dt$ (signed area). **Distance** = $\int|v|\,dt$: split integral at roots of v = 0, then add up the magnitudes.

When a Depends on v or x

Given	Separate	Result
a(t)	dv = a dt	v(t), then x(t)
a(v)	dv/a(v) = dt	v(t)
a(x)	v dv = a dx	v(x)

Chain rule: $a = dv/dt = (dv/dx)(dx/dt) = v\,dv/dx$. Use when a = a(x); then $\int v\,dv = \int a\,dx$ gives v as a function of x.

Worked example: $a = -kv \to \int dv/v = -k\int dt \to v = v_o e^{-kt}$. v → 0 but never actually reaches it; x → $x_o + v_o/k$.
Limits: use definite integrals w/ matching limits ($v_o \to v$ on left, 0→t on right) to skip solving for +C later.
Worked: a = 6t, v_o = 2 m/s, x_o = 0 → $v = 3t^2 + 2$, $x = t^3 + 2t$.
Distance example: v = t − 2 on 0–4 s → Δx = 0 m, but distance = 2 + 2 = 4 m (split at t = 2 s).

Average Value

$\mathbf{v}_{avg} = (1/\Delta t)\int v\,dt = \Delta x/\Delta t$. $\mathbf{(v_i + v_f)/2}$ works ONLY for const a.
Graph link: $\int a\,dt$ = area under a-t = Δv; $\int v\,dt$ = area under v-t = Δx. Slopes reverse each step (x-t slope = v, v-t slope = a).
Check: differentiate your x(t) back; it must return the given v(t), and x(0) must equal x_o. Do this every time.
Exponentials: $\int e^{-kt}dt = -(1/k)e^{-kt}$; $\int dv/v = \ln v$.

⚠ Don't drop +C. A missing x_o or v_o is a classic lost point on derivation FRQs; write it explicitly.

✓ Check units after integrating: $\int(m/s^2)\,dt \to m/s$. A mismatch means a setup error somewhere.

1.2 Constant Acceleration & Free Fall

The Big 4 (const a ONLY)

Equation	Missing var
$v = v_o + at$	Δx
$\Delta x = v_o t + \frac{1}{2}at^2$	v
$v^2 = v_o^2 + 2a\Delta x$	t
$\Delta x = \frac{1}{2}(v_o + v)t$	a

Pick the eq: list 5 vars (Δx, v_o, v, a, t); choose the eq missing the one you don't know & don't need.
Origin: integrate a = const once → v(t); twice → x(t). Know how to derive them for 'derive' FRQs, step by step.
Worked example: car at 20 m/s brakes evenly at 5 m/s² → stops in t = 4 s, $\Delta x = v_o^2/2a = 40$ m (v^2 eq, no t needed).
Stopping distance ∝ v_o^2: double the speed → 4× the distance (at the same constant braking deceleration).

Free Fall

Free fall: gravity only; a = g ≈ 9.8 m/s² down, same for all masses (no air resistance), even at the peak.
Symmetric vertical toss: $t_{up} = t_{down}$; returns to the hand at launch speed; $v_{top} = 0$; $h_{max} = v_o^2/2g$; $t_{up} = v_o/g$.
Sign choice: up = + → a = −g. Down = + → a = +g. Either works if you stay consistent for the entire problem.
Dropped from rest: $\Delta y = \frac{1}{2}gt^2$, $v = gt$, $v^2 = 2g\Delta y$ (taking down as +). Distances fallen in each 1 s step grow 1:3:5.

Validity Check

Is a constant? If a depends on t, v, or x → the Big 4 are invalid; integrate instead (separate the variables first).
Two objects: write x(t) for each w/ same origin & clock; set $x_1 = x_2$ to find when they meet; plug t back for where.

⚠ Using $v = v_o + at$ when a = a(t) is the #1 Physics C kinematics error. Check a first, every time.

✓ Quadratic in t gives 2 roots: keep the one that fits (t ≥ 0, right direction). Negative root = before launch.

1.3 Representing Motion: Graphs

Slopes & Areas

Graph	Slope =	Area =
x vs t	velocity	(not used)
v vs t	acceleration	Δx
a vs t	(jerk)	Δv

x-t curvature: concave up → a > 0; concave down → a < 0; straight line → a = 0 (constant velocity, no curve).
v-t crosses zero = turning point. Area above axis = +Δx; below axis = −Δx. Distance adds both as positives.
Constant a: x-t is a parabola, v-t a sloped line, a-t a horizontal line. Nonconstant a → curved v-t, so Big 4 fail.

Translating Representations

Graph → eq: read intercept (initial value) + slope. **Eq → graph**: mark zeros, max/min, concavity.
Motion diagram: dots **a**t equal Δt; spacing grows = speeding up. Draw **v** & a arrows at each dot to show direction.
Sketch rules: x & v are continuous (no jumps); a can jump. Label axes, units, key points, intercepts, and peaks.

Reading Values

Instantaneous v on x-t: draw the tangent line, then rise/run.
Avg v: secant slope between 2 points on the curve.
Counting area: split into rectangles & triangles; for curves, count grid boxes × area per box. Below axis = −.
Worked example: v-t line from 4 to −4 m/s over 4 s → a = −2 m/s², Δx = 0, distance = 8 m (it turns around at t = 2 s).
a-t → v-t: const a segment → straight v-t segment; a = 0 → flat v-t; a jump in a → kink in v-t; area under a-t = Δv.
Kink in x-t = sudden jump in v (idealized bounce or collision).

⚠ Height on a v-t graph is NOT position. Get x from area (plus x_o), not by reading the y-value.

✓ Translation FRQ: every feature of your graph (slope, zero, kink, curvature) must agree w/ the eq or data.

1.4 Reference Frames & Relative Motion

Core Ideas

Reference frame: observer's coordinate system + clock. Position & velocity depend on frame (ground vs. train).
Inertial frame: not **a**ccelerating (v_{frame} = const). All inertial frames measure the same a (and forces).

Relative Velocity

Addition: $\mathbf{v}_{A/C} = \mathbf{v}_{A/B} + \mathbf{v}_{B/C}$. Inner subscripts (B) "cancel."
Reverse order flips sign: $\mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$. **Position**: $\mathbf{r}_{A/C} = \mathbf{r}_{A/B} + \mathbf{r}_{B/C}$; take d/dt → get the velocity addition rule above.
Relative accel: if frame B moves at const v, $\mathbf{a}_{A/C} = \mathbf{a}_{A/B}$.

River & Wind Problems

Goal	How
Velocity vs shore	v(b/s) = v(b/w) + v(w/s)
Cross in min time	Head straight across
Land straight across	Aim up: sinθ = v(w)/v(b)
Crossing time	t = width / $v_\perp$

Plane in wind: same setup; air = water, ground = shore.

Classic Scenarios

Rain on car: $\mathbf{v}_{rain/car} = \mathbf{v}_{rain/gnd} - \mathbf{v}_{car/gnd}$ → rain looks slanted toward the driver; faster car = more slant.
Ball tossed on train: rider sees vertical line; ground sees parabola. Both see a = g (inertial frames).
Example: boat 4 m/s (rel. water) aimed across a 3 m/s current → 5 m/s, 37° downstream. 120 m wide → 30 s.
Same-direction cars: A at 30 m/s, B at 20 m/s → A gains on B at 10 m/s; opposite directions → 50 m/s.

⚠ Switching to a moving observer means subtracting: v(A rel. B) = v(A) − v(B), always as vectors (draw it).

✓ Draw the vector triangle; label each arrow w/ its full subscript (like A/B) before writing any equation.

1.5 Motion in 2D & 3D: Projectiles

Independence of Axes

Axes are independent: x & y share only t. Solve each axis w/ its own ($a_x = 0$, $a_y = -g$ for projectiles).
Vector form: $\mathbf{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$; $\mathbf{v} = d\mathbf{r}/dt$, $\mathbf{a} = d\mathbf{v}/dt$, component by component, then recombine.

Projectiles (no air)

	x	y
a	0	−g
v(t)	$v_o\cos\theta$	$v_o\sin\theta - gt$
pos(t)	$v_o\cos\theta \cdot t$	$v_o\sin\theta \cdot t - \frac{1}{2}gt^2$

Range (level ground): $R = v_o^2\sin 2\theta/g$; max at 45°.
Complementary angles (30°/60°) → same R.
Peak: $v_y = 0$ but $v = v_x \neq 0$. **Flight time** (level): $T = 2v_o\sin\theta/g$.
Path: eliminate t → $y = x\tan\theta - gx^2/(2v_o^2\cos^2\theta)$, a parabola.
Horizontal launch: $v_{0y} = 0$, so fall time depends only on launch height: $t = \sqrt{(2h/g)}$; horizontal range = $v_o t$.
Level-ground landing: impact speed = launch speed; impact angle = θ below horizontal (mirror image of launch).

General 2D/3D Motion

Given r(t): differentiate each component; **speed** = $\sqrt{v_x^2 + v_y^2 + v_z^2}$; direction from components ($\tan^{-1} v_y/v_x$).
Parts of a: component ∥ **v** alters speed; component ⊥ v alters direction (a curved path, e.g., circular motion).
Worked example: 20 m/s at 30° → v_x = 17.3, v_{0y} = 10 m/s; T = 2.04 s; R ≈ 35.3 m. 60° gives the same R (complementary).
Incline: tilt axes along the slope only if it helps; g then splits neatly into g sinθ (along slope) + g cosθ (⊥ to slope).

⚠ $R = v_o^2\sin 2\theta/g$ only if launch & landing heights match. Otherwise solve $y(t) = y_{land}$ for the flight time t first.

⚠ At the very peak, a = g (not 0) and speed = the horizontal speed $v_o\cos\theta$ (not 0). Only $v_y = 0$ at the top.

Unit 1 Exam Moves (MCQ & FRQ)

FRQ Types & Kinematics

FRQ type	Unit 1 angle
Math Routines	Derive v(t), x(t) from a
Translation	Eq ↔ graph ↔ words
Exp. Design	Measure g or ramp a
Qual/Quant	Predict, then justify

Derivations

Start from a definition (a = dv/dt) or a given eq; show each step; final answer in given variables only.
"Derive" ≠ "calculate": answer in given symbols (v_o, g, k, θ), no plugged-in numbers or units mixed in at all.
Justify: cite the physics ("area under v-t = Δx"), not just "the graph shows." Name the principle or definition used.

Lab & Data

Linearize: from rest, $\Delta x = \frac{1}{2}at^2$ → plot Δx vs t^2; slope = a/2. Plot v^2 vs Δx → slope = 2a (from rest, both linear).
Tools: motion sensor, photogates, video analysis. Repeat trials & average to cut random uncertainty.
Free-fall lab: drop from several heights, time w/ photogate; plot h vs t^2 → slope = g/2, so g = 2 × slope.
Best-fit line: follows the trend, not dot-to-dot. Take slope from 2 far-apart points on the line, not data.

MCQ Traps

Limiting cases: test choices w/ θ = 0, θ = 90°, g → 0. **Units**: cut answer choices w/ wrong dimensions.
Units & sig figs: always write units; ~3 sig figs on final answers. Don't round in the middle of a calculation.

✓ The equation sheet lists the kinematics eqs. Your points come from choosing the right one & setting up integrals.

⚠ Read "speed" vs "velocity" and "distance" vs "displacement" in every question stem.