The Simple AP® Precalculus Unit 1 Cheat Sheet

1.1–1.3 Change in Tandem & Rates of Change

1.1 Change in Tandem

Function: each input → exactly 1 output. Input & output vary together (in tandem).
Increasing: x↑ → f(x)↑. **Decreasing**: x↑ → f(x)↓. State intervals in x-values, not y.
Concave up: rate of change ↑ (graph bends up).
Concave down: rate of change ↓.
Zeros: inputs where f(x) = 0 → x-intercepts of the graph; f can change sign there.

1.2 Average Rate of Change

AROC (average rate of change) on [a, b] = [f(b) − f(a)] / (b − a) = slope of the **secant line** thru (a, f(a)) & (b, f(b)); units = output per input.
Rate at a point: approx. w/ AROC over shorter & shorter intervals that contain the point.
Sign: rate of change > 0 → f increasing; rate < 0 → f decreasing on that whole interval.

1.3 Linear vs. Quadratic

Function	Over equal Δx	Graph
Linear	1st diffs const	No concavity
Quadratic	2nd diffs const	Concave 1 way
Cubic	3rd diffs const	Can switch

Example: f(x)=x² at x=0,1,2,3 → 0,1,4,9 → 1st diffs 1,3,5 → 2nd diffs 2,2 → quadratic.

⚠ AROC = Δoutput ÷ Δinput. Never average the two outputs; that's a mean, not a rate.

✓ Table Qs: check that every Δx is equal before you use 1st or 2nd differences.

1.4 Polynomial Functions & Rates of Change

Form & Vocabulary

Polynomial: $p(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, w/ whole-number powers (0, 1, 2…) only.
Degree n = highest power of x. **Leading coeff.** $a_n \neq 0$. Constant a_0 = the y-intercept, p(0).

Extrema

Local max: f switches inc → dec. **Local min**: dec → inc. Rate of change switches sign there.
Global (absolute) max/min: the greatest/least output value over the function's whole domain.
Even degree → has a global max OR global min (never both). **Odd degree** → neither.
Turning pts: cubic → 0 or 2; quartic → 1 or 3; degree n → at most n − 1 in total.

Inflection & Concavity

Point of inflection: rate of change switches inc ↔ dec → concavity flips (up ↔ down).

Feature	Graph	Rate of change
Local max	Peak	+ → −
Local min	Valley	− → +
Concave up	Bends up	Increasing
Inflection pt	Bend flips	Max/min rate

⚠ Decreasing + concave up ≠ increasing. Concavity is about the rate, not the outputs.

✓ Justify extrema in words: "f changes from increasing to decreasing at x = 2."

1.5 Polynomial Functions & Complex Zeros

Zeros, Factors, Intercepts

Real zero a ⟺ (x − a) is a factor ⟺ (a, 0) is an x-intercept of the polynomial's graph.
Fundamental Thm of Algebra: degree n → exactly n complex zeros (count repeats = multiplicity).
Non-real zeros come in conjugate pairs: a + bi → a − bi too (when all the coefficients are real).
Graph shows real zeros only; non-real zeros never appear as x-intercepts on the graph.
Example: $x^3 - x^2 + 9x - 9 = x^2(x - 1) + 9(x - 1) = (x - 1)(x^2 + 9)$ → zeros 1, ±3i (grouping).

Multiplicity

Multiplicity	At x-axis	Sign of f
1 (odd)	Crosses	Switches
2, 4 (even)	Touches, bounces	Same
3, 5 (odd ≥3)	Crosses, flattens	Switches

Build from zeros: 2 (mult 2), −1, 3i → $p(x) = a(x - 2)^2(x + 1)(x^2 + 9)$. Use a pt to find a.

Inequalities & Symmetry

Sign chart: between consecutive real zeros, p stays all + or all −. Test 1 x in each interval.
Even fn: f(−x) = f(x) → symmetric over y-axis. **Odd fn**: f(−x) = −f(x) → symmetric about origin.
Ex: x^3 & x^5 are odd fns; $x^2 + 1$ & x^4 are even fns.
Example: (x + 1)(x − 3) < 0 → test −2 (+), 0 (−), 4 (+) → so the solution set is −1 < x < 3.

✓ p(x) ≥ 0: include the zeros themselves in the answer; for p(x) > 0, exclude them.

1.6 Polynomials & End Behavior

Leading Term Rules

End behavior: as |x| gets large, the **leading term** a_nx^n dominates p(x), so the lower-degree terms barely matter. **LC** = the leading coefficient.

Degree, LC	x → −∞	x → ∞
Even, LC > 0	p → ∞	p → ∞
Even, LC < 0	p → −∞	p → −∞
Odd, LC > 0	p → −∞	p → ∞
Odd, LC < 0	p → ∞	p → −∞

Limit Notation

Read: $\lim_{x\to\infty} p(x) = \infty$ means "as x increases w/o bound, p(x) increases w/o bound."
Example: $p(x) = -2x^3 + 5x^2 + 1$ → odd, $a_n < 0$ → $\lim_{x\to-\infty} = \infty$ & $\lim_{x\to\infty} = -\infty$ (opposite ends).
Factored form: multiply leading terms. $-x(x - 1)^2(x + 4)$ → $-x^4$ → even, LC < 0 → both ends ↓.

Connecting Features

Odd degree: ends go opposite ways → at least 1 real zero; its range = all real numbers.
Even degree: ends match → may have 0 real zeros (e.g., $x^2 + 1$ never touches the x-axis).
Even degree, $a_n > 0$ → a global min exists; $a_n < 0$ → a global max exists instead (never both).

⚠ End behavior ignores lower terms only for large |x|. Near 0, they matter a lot.

✓ MCQ graph match: check ends + y-intercept first; that kills most choices fast.

1.7–1.8 Rational Functions: End Behavior & Zeros

Compare Degrees (1.7)

Rational fn: r(x) = p(x)/q(x), polynomials, q(x) ≠ 0. Ends (horizontal asymptote, HA) follow the ratio of the leading terms of p & q as |x| → ∞.

deg p vs. q	End behavior	Example
p < q	HA y = 0	$3x/(x^2+1)$
p = q	HA y = a/b	$2x^2/(x^2-9)$
p = q + 1	Slant asym.	$(x^2+1)/x$
p > q + 1	Like a poly	$x^4/(x-1)$

Slant asymptote: long divide; quotient line = asymptote. $(x^2 + 1)/x = x + 1/x$ → y = x.
Ex: $2x^2/(x^2 - 9)$ → equal degrees → HA y = 2/1 = 2.

Zeros (1.8)

Zeros of r: real zeros of p that are in r's domain (not also zeros of q, which make holes).
Example: $r(x) = (x^2 - 4)/(x^2 - x - 6) = (x-2)(x+2)/[(x-3)(x+2)]$ → only zero x = 2.
Domain of r: all reals except the zeros of q.

Rational Inequalities

Sign chart: mark zeros of p AND q; sign can switch at each. Never include zeros of q.
Example: (x − 1)/(x + 2) ≥ 0 → x < −2 or x ≥ 1 (−2 excluded b/c q = 0 there; 1 included).

⚠ Zero of numerator that also zeros the denominator = hole, not an x-intercept.

✓ Factor numerator & denominator fully before naming any zero, asymptote or hole.

1.9–1.10 Vertical Asymptotes & Holes

Vertical Asymptotes (1.9)

VA (vertical asymptote) at x = a: a is a zero of q w/ higher multiplicity in q than in p.
Near a VA: $\lim_{x\to a^-} r(x)$ & $\lim_{x\to a^+} r(x)$ are each ∞ or −∞: r is unbounded on both sides.
Side behavior: odd mult. in q (after cancel) → opposite signs each side; even → same sign.

Holes (1.10)

Hole at x = a: a is a zero of p & q w/ multiplicity in p ≥ multiplicity in q (factor cancels fully).
Hole's y: cancel common factor, plug a into simplified r. $\lim_{x\to a} r(x)$ = that value.

Zero a of q	Feature	Limit at a
Not a zero of p	VA	±∞
Mult. q > mult. p	VA	±∞
Mult. p ≥ mult. q	Hole	Finite

Worked Example

r(x) = $(x^2 - 4)/(x^2 - x - 6)$ → simplified (x − 2)/(x − 3), w/ x ≠ −2 kept from the original form.
Features: hole (−2, 4/5); VA x = 3; zero x = 2; HA y = 1 (equal degrees); y-intercept (0, 2/3).
Domain: x ≠ −2 & x ≠ 3 → every zero of q is excluded, whether it becomes a VA or a hole.

⚠ Canceling does not fix the domain: x = −2 is still excluded, even after you simplify.

⚠ A graph never crosses a VA, but it can cross an HA (an HA describes end behavior only).

1.11–1.12 Equivalent Forms & Transformations

Pick the Right Form (1.11)

Standard form → degree, end behavior, y-int.
Factored form → zeros, VAs, holes.
Long division: p(x) = d(x)·q(x) + r(x), w/ deg r < deg d. It reveals slant asymptotes of rational fns.
Binomial Thm: $(a + b)^n$ coeffs = row n of **Pascal's triangle** (1 3 3 1 for n = 3; 1 4 6 4 1 for n = 4).
Example: $(x + 2)^3 = x^3 + 3(x^2)(2) + 3(x)(4) + 8 = x^3 + 6x^2 + 12x + 8$ (row 3 coeffs: 1 3 3 1).

g(x) = a·f(b(x − h)) + k (1.12)

Parameter	Effect on graph
a (outside)	Vert. dilation by |a|
a < 0	Reflect over x-axis
k (outside)	Shift up k
b (inside)	Horiz. dilation 1/|b|
b < 0	Reflect over y-axis
h (inside)	Shift right h

Inside vs. outside: inside changes act on x (and feel reversed); outside act on y.
Map a point: (x, y) on f → (x/b + h, ay + k) on g.
Example: g(x) = −2f(x − 1) + 3 → right 1, vert. stretch ×2, reflect over x-axis, up 3.

⚠ f(x + 3) shifts LEFT 3. f(2x) squeezes the graph toward the y-axis by a factor of 1/2.

✓ Factor b first: f(2x − 6) = f(2(x − 3)) → shift right 3 (not 6) & dilate horizontally by 1/2.

1.13–1.14 Function Model Selection & Construction

Pick a Model (1.13)

Model	Data clue
Linear	Const rate of change
Quadratic	Rates change linearly
Cubic / higher	Many extrema, inflect.
Rational	Asymptote, y ∝ 1/x
Piecewise	Rule shifts by interval

Geometry clues: area → quadratic (2 dims); volume → cubic (3 dims, like a box).

Assumptions & Restrictions

Domain: limit to inputs that make sense (time ≥ 0, whole-number items, box sides > 0).
Range: outputs must make sense too (no negative counts, lengths, costs or times).

Build & Use a Model (1.14)

From points: plug into $y = ax^2 + bx + c$ → solve the system (or calculator regression).
Interpret: give each parameter & each answer w/ units in context (e.g., "dollars per item").
Apply: predict output, or solve model = target for input. Check answer is in the domain.
Rational example: avg cost C(x) = (500 + 2x)/x → HA y = 2 → cost per item → $2 as x grows.

✓ FRQ 2 (modeling): show the setup, then round to 3 decimal places w/ units in context.

⚠ Models break outside the data range: flag any extrapolation as much less reliable.