The Simple AP® Precalculus Unit 2 Cheat Sheet

2.1–2.2 Sequences & Rates of Change

Arithmetic vs. Geometric

	Arithmetic	Geometric
Pattern	Add d	Multiply by r
Formula	$a_n = a_0 + dn$	$g_n = g_0 r^n$
From term k	$a_k + d(n - k)$	$g_k r^{n-k}$
Function type	Linear	Exponential

Common difference d = $a_{n+1} - a_n$. **Common ratio** r = g_{n+1}/g_n; same for every consecutive pair.

Example: $g_2 = 12$ & $g_5 = 96 \rightarrow r^3 = 96/12 = 8 \rightarrow r = 2$, so $g_0 = 12/4 = 3$ & the formula is $g_n = 3 \cdot 2^n$.

Linear vs. Exponential Change

Linear: over equal-length input intervals, output values change by equal DIFFERENCES (additive).
Rate of change: linear rate is constant; exponential rate grows w/ the function's value.
Exponential: over equal-length input intervals, outputs change by equal RATIOS.
Table check: inputs must be equally spaced. Find successive differences, then successive ratios.

Connections

⚠ Ratios only mean exponential when the inputs are equally spaced. Check the x-column first.

✓ FRQ: "Over equal-length input intervals, outputs change by a constant ratio, so f is exponential." Use these words.

2.3–2.4 Exponential Functions

General Form $f(x) = ab^x$

Conditions: a ≠ 0 (initial value, f(0) = a), b > 0 & b ≠ 1 (base = growth or decay factor per unit of x).

a, b	Behavior	Concavity
a > 0, b > 1	Increasing	Concave up
a > 0, 0 < b < 1	Decreasing	Concave up
a < 0, b > 1	Decreasing	Concave down
a < 0, 0 < b < 1	Increasing	Concave down

Key features: domain all reals; range y > 0 if a > 0; horizontal asymptote y = 0; always increasing or always decreasing; no max/min.

Natural base e ≈ 2.718; e^x models continuous growth. $f(x) = ab^x + k \rightarrow$ asymptote y = k.
Growth vs. rate: b = 1.08 → 8% growth per unit of x; b = 0.92 → 8% decay per unit of x.

Exponent Rules as Transformations

Product: $b^{x+k} = b^k \cdot b^x$, so a horizontal shift is the same as a vertical dilation by b^k.

Power: $(b^m)^n = b^{mn} \rightarrow b^{cx} = (b^c)^x$: a horizontal dilation changes the base from b to b^c.

Example: $3 \cdot 2^{x+4} = 3 \cdot 16 \cdot 2^x = 48 \cdot 2^x$, so a shift left 4 is the same as a stretch by 16.

Negative exponent: $b^{-x} = (1/b)^x$; reflecting over the y-axis turns growth into decay.

⚠ $ab^x \neq (ab)^x$: only b gets the exponent.

2.5–2.6 Exponential Modeling & Validation

Building Models

Growth/decay rate r: $f(t) = a(1 + r)^t$; growth → b = 1 + r, decay → b = 1 − r (r as a decimal).

Context	Model
Grows 6% per yr	$a(1.06)^t$
Doubles every 5 hr	$a \cdot 2^{t/5}$
Half-life 30 yr	$a(½)^{t/30}$
Compounded n×/yr	$P(1 + r/n)^{nt}$
Continuous	Pe^{rt}

From 2 points: (0, 50) & (3, 400) → a = 50, $b^3 = 8 \rightarrow b = 2$. Use ratios, then solve for b & a.

Equivalent forms: $2^{t/5} = (2^{1/5})^t \approx 1.149^t$, about 14.9% growth per hour (same model).

Validating a Model

Residual = actual − predicted. Positive → model underestimates; negative → model overestimates.
Residual plot: random scatter around 0 → good model. A clear pattern → try another model.
Choosing a model: equal differences → linear; equal ratios → exponential; neither → recheck.
Assumptions: models only work over a reasonable domain; growth can't go on forever.

⚠ Residual = actual − predicted. Its sign tells you whether the model over- or underestimates.

✓ Interpret b in context: "the population is multiplied by 1.06 each year" (+6%/yr).

2.7–2.8 Composition & Inverse Functions

Composition

f(g(x)): output of g becomes input of f. Work inside out: find g(x) first, then apply f to that output.
Order matters: f(g(x)) ≠ g(f(x)) in general. $f(x) = x^2$, $g(x) = x + 1 \rightarrow (x + 1)^2$ vs. $x^2 + 1$.
From tables or graphs: f(g(2)): read g(2), then find f at that value. Input must be in f's domain.
Decomposition: $h(x) = \sqrt{(3x - 1)} = f(g(x))$ w/ $g(x) = 3x - 1$ (inner) & $f(x) = \sqrt{x}$ (outer function).

Inverse Functions

f^{-1} undoes f: $f(f^{-1}(x)) = x$ & $f^{-1}(f(x)) = x$. If f(a) = b, then $f^{-1}(b) = a$ (swap input & output).
Find it: write y = f(x), swap x & y, solve for y. $f(x) = 2x - 6 \rightarrow x = 2y - 6 \rightarrow f^{-1}(x) = (x + 6)/2 = x/2 + 3$.
Domain & range: f^{-1}'s domain is f's range, & f^{-1}'s range is f's domain (they swap).
Graphs: f & f^{-1} are reflections over the line y = x.

f	f^{-1}
Domain	Range of f
Range	Domain of f
Point (a, b)	Point (b, a)
Graph	Reflect over y = x

One-to-one: each output comes from exactly 1 input (horizontal line test); else restrict.

⚠ $f^{-1}(x) \neq 1/f(x)$. The −1 means the inverse function, not a reciprocal of f(x).

2.9–2.10 Logarithmic Expressions

What a Log Means

Definition: $\log_b c = a \Leftrightarrow b^a = c$. A log asks "what power of b gives c?" ($\log_3 81 = 4$ b/c $3^4 = 81$).

Restrictions: b > 0, b ≠ 1, c > 0. Logs of 0 or of negative numbers are undefined.

Expression	Value	Why
$\log_2 8$	3	$2^3 = 8$
log 1000	3	$10^3 = 1000$
$\log_5 (1/25)$	−2	$5^{-2} = 1/25$
$\log_b 1$	0	$b^0 = 1$
$\log_b b$	1	$b^1 = b$
$\ln e^4$	4	Base e

Common log: $\log x = \log_{10} x$. **Natural log**: $\ln x = \log_e x$, where e ≈ 2.718 (calculator "ln").

Estimate: $\log_2 20$ is between 4 & 5 ($16 < 20 < 32$).

Logs Undo Exponentials

Inverses: $y = b^x$ & $y = \log_b x$ undo each other, so $\log_b(b^x) = x$; their graphs are reflections of each other over the diagonal line y = x.

Graphs: $y = \log_b x$ is $y = b^x$ reflected over y = x; horizontal asymptote ↔ vertical asymptote.

⚠ A log's value is an EXPONENT: $\log_2 8 = 3$.

2.11–2.12 Logarithmic Functions

Graph of $f(x) = a \log_b x$

Feature	$f(x) = \log_b x$
Domain	x > 0
Range	All real numbers
Asymptote	Vertical, x = 0
x-intercept	(1, 0)
Key point	(b, 1)

b > 1, a > 0: increasing & concave down. **0 < b < 1**: decreasing & concave up. Always 1 direction.

End behavior (b > 1): as $x \rightarrow 0^+$, $f(x) \rightarrow -\infty$; as $x \rightarrow \infty$, $f(x) \rightarrow \infty$, but more & more slowly.
Shifts: $\log_b(x - h) \rightarrow$ asymptote x = h.

Power rule as a transformation: $\log_b(x^n) = n \log_b x$, a vertical dilation by a factor of n.

Log Properties

Rule	Statement
Product	$\log_b(xy) = \log_b x + \log_b y$
Quotient	$\log_b(x/y) = \log_b x - \log_b y$
Power	$\log_b(x^n) = n \log_b x$
Change of base	$\log_b x = \ln x / \ln b$

⚠ $\log(x + y) \neq \log x + \log y$. The rules only work for products & quotients inside a log.

✓ Condense: $2 \log x - \log 5 = \log(x^2/5)$. Expand: $\ln(x^3y) = 3 \ln x + \ln y$. Practice both directions.

2.13 Exponential & Log Equations

Exponential Equations

Same base: $4^{x+1} = 8^x \rightarrow 2^{2x+2} = 2^{3x} \rightarrow x = 2$.

Take a log: $5 \cdot 3^x = 40 \rightarrow 3^x = 8 \rightarrow x = \log_3 8 = \ln 8 / \ln 3 \approx 1.893$ (change of base on a calculator).

Base e: $2e^{0.4t} = 18 \rightarrow e^{0.4t} = 9 \rightarrow t = \ln 9/0.4$.
Isolate first: divide by a & subtract k before taking the log of both sides ($5 \cdot 3^x = 40 \rightarrow 3^x = 8$).

Switch forms: $e^x = 5 \Leftrightarrow x = \ln 5$; $\ln x = 2 \Leftrightarrow x = e^2 \approx 7.39$ (use the calculator for decimals).

Logarithmic Equations

Rewrite: $\log_2(x - 3) = 4 \rightarrow x - 3 = 16 \rightarrow x = 19$.

Log = log: $\log_b M = \log_b N \rightarrow M = N$, b/c logs are one-to-one; then check M & N are both > 0.
Combine first: $\log x + \log(x - 3) = 1 \rightarrow x(x - 3) = 10 \rightarrow x = 5$ or $x = -2$; only x = 5 works.
Check the domain: x = −2 makes log x undefined, so the only solution is x = 5 (extraneous root).

Inequalities & Inverses

Inequalities: base b > 1 keeps the sign; 0 < b < 1 flips it. $(½)^x > 8 \rightarrow 2^{-x} > 2^3 \rightarrow -x > 3 \rightarrow x < -3$.
Graph to check: solve f(x) = c by finding where y = f(x) meets the line y = c on a calculator.

⚠ Always plug log solutions back in. A value that makes a log's input ≤ 0 is extraneous.

✓ No-calculator FRQ: give exact answers like ln 9 / 0.4; don't round unless the question asks.

2.14–2.15 Log Modeling & Semi-log Plots

Logarithmic Models

When to use: output grows quickly at first, then increases more & more slowly (concave down).
Form: $f(x) = a + b \ln x$ (calculator log regression).
Pattern: output rises by the same amount each time the input is multiplied by the same factor.
Log scales: pH & Richter: each step of 1 = ×10; decibels: each +10 dB = ×10 sound intensity.
Example: pH 3 vs. pH 5 → 100 times more acidic.

Semi-log Plots

Semi-log plot: y-axis on a log scale (1, 10, 100: equal spacing = ×10); x-axis is linear.

Key fact: exponential data $y = ab^x$ appears as a straight LINE on a semi-log plot (log y vs. x).

Linear model	Means
$\log y = (\log b)x + \log a$	Line in x & log y
Slope	log b
Vertical intercept	log a

Back out the model: slope 0.3 & intercept 1 (base 10) → $b = 10^{0.3} \approx 2$, $a = 10^1 = 10 \rightarrow y \approx 10 \cdot 2^x$.
Linearize: take log of each y-value; if (x, log y) is a line, an exponential model fits the original data.

Picking a Model

⚠ On a semi-log plot, the slope is log b, not b. Undo the log to get b ($b = 10^{\text{slope}}$).

✓ Straight line on a semi-log plot → exponential model fits. Curve → it doesn't.