The Ultimate Digital SAT® Math Algebra Cheat Sheet

Linear Equations in 1 Variable

Solving

Goal: isolate x. Distribute, combine like terms, move x terms to 1 side, then divide by the coefficient.
Fractions: multiply every term by the LCD first: $x/3 + x/4 = 7 \rightarrow 4x + 3x = 84 \rightarrow x = 12$. Fewer fraction errors.
Decimals: multiply by 10 or 100 to clear them: $0.2x + 1.5 = 3.1 \rightarrow 2x + 15 = 31 \rightarrow x = 8$ (check: 1.6 + 1.5).
Variables on both sides: $7x - 4 = 3x + 12 \rightarrow 4x = 16 \rightarrow x = 4$. Subtract the smaller x-term to avoid negatives.
Distribute carefully: $-3(2x - 5) = -6x + 15$. Dropping the sign on the 2nd term is the #1 trap in algebra.

How Many Solutions?

Simplifies to	Solutions	Example
x = number	Exactly 1	3x + 2 = 11
a = a (true)	Infinitely many	2(x + 3) = 2x + 6
a = b (false)	None	2x + 1 = 2x + 5

Find k for no solution: $kx + 4 = 3x - 1 \rightarrow$ set k = 3 (same x-coefficient), since the constants 4 & −1 differ.
Infinitely many: both the x-coefficients AND the constants must match on each side after simplifying.
Example: $3(x + 2) = 3x + c$ has infinitely many solutions when c = 6, & no solution for any other value of c.

Solve for an Expression

Shortcut: if $4x + 6 = 18$, then $2x + 3 = 9$ (divide both sides by 2). Don't solve for x first; it wastes time.
Literal equations: solve for a letter: $P = 2l + 2w \rightarrow w = (P - 2l)/2$. Treat the other letters as numbers.
Example: $5(x - 2) = 3x + 4 \rightarrow 5x - 10 = 3x + 4 \rightarrow 2x = 14 \rightarrow x = 7$. Check: 5(5) = 25 & 3(7) + 4 = 25 ✓.

⚠ Read what's asked: "What is 3x − 1?" The answer choice w/ just the value of x is a planted trap.

✓ Desmos check: type each side as y = left & y = right; the intersection's x-value is the solution.

Linear Functions

Function Basics

Linear function: f(x) = mx + b. m = slope (rate of change), b = f(0), the y-intercept or starting value.
Input vs. output: x is the input (independent variable); f(x) is the output (dependent variable, plotted as y).
Notation: f(3) means the output when x = 3. If $f(x) = 2x - 5$, then $f(3) = 1$ & $f(x + 1) = 2(x + 1) - 5 = 2x - 3$.
Solving f(x) = k: set the expression equal to k: $2x - 5 = 9 \rightarrow x = 7$, so f(7) = 9; the input is the answer.
Working backward: if $f(x) = 3x + 2$ & f(a) = 17, then $3a + 2 = 17 \rightarrow 3a = 15 \rightarrow a = 5$. The answer is an input.

From a Table or 2 Points

x	1	3	5
f(x)	4	10	16

Slope: $m = \Delta y/\Delta x = (10 - 4)/(3 - 1) = 3$. **Intercept**: $4 = 3(1) + b \rightarrow b = 1$, so $f(x) = 3x + 1$. Check: f(5) = 16.
Linear check: equal steps in x must give equal changes in y; otherwise the function isn't linear (could be exponential).
2 points given: (2, 7) & (6, 15) → m = 8/4 = 2 → 7 = 2(2) + b → b = 3 → f(x) = 2x + 3. Check (6, 15): 12 + 3.

Transformations

f(x) + k shifts up k; **f(x – h)** shifts right h. For lines, shifts change the y-intercept, not the slope.
Graph reading: f(a) is the y-value at x = a; f(x) = 0 at the x-intercept, where the line crosses the x-axis.
Direct variation: y = kx is a line through (0, 0); y/x stays constant, so doubling x doubles y (k = y/x).
Rate of change: a linear function changes by a constant amount (m) for each 1-unit increase in x.
Increasing vs. decreasing: m > 0 → f rises as x grows; m < 0 → f falls; m = 0 → f is constant (a flat line).

⚠ $f(x + 1) \neq f(x) + 1$. Substitute (x + 1) for every x in the rule, then simplify. Same idea for f(2x) & f(−x).

✓ Table question? Type the points into a Desmos table & run a linear regression: $y_1 \sim mx_1 + b$ gives m & b.

Linear Equations in 2 Variables

Slope

Slope $m = (y_2 - y_1)/(x_2 - x_1)$ = rise/run. Positive → rises; negative → falls; 0 → flat; bigger |m| → steeper.
Vertical line x = c has undefined slope; **horizontal line** y = c has slope 0. Only vertical lines can't fit y = mx + b.

Forms of a Line

Form	Equation	Shows
Slope-int.	$y = mx + b$	Slope, y-int
Point-slope	$y - y_1 = m(x - x_1)$	1 point
Standard	$Ax + By = C$	Intercepts

Standard form shortcuts: slope = −A/B; x-intercept = C/A; y-intercept = C/B. Faster than rewriting the equation.
Example: $3x + 4y = 24 \rightarrow$ slope −3/4, x-intercept (8, 0), y-intercept (0, 6). No rearranging needed.

Parallel & Perpendicular

Parallel: same slope, different intercepts. **Perpendicular**: slopes are negative reciprocals ($m_1 \cdot m_2 = -1$).
Example: perpendicular to $y = (2/3)x + 1$ through (0, 4) → slope $-3/2 \rightarrow y = -(3/2)x + 4$. Product check: −1.
On the line? A point lies on a line only if its coordinates make the equation true: test (x, y) by plugging in.
Missing value: if (k, 5) is on $y = 2x - 3$, then $5 = 2k - 3 \rightarrow k = 4$. Plug in the coordinate you already know.
Intercepts: set y = 0 to find the x-intercept; set x = 0 to find the y-intercept. Both are points, e.g., (8, 0).
Line from 2 points: (1, 3) & (4, 12) → m = 9/3 = 3 → y − 3 = 3(x − 1) → y = 3x, which passes through (0, 0).
Same line? $2x + 3y = 6$ & $4x + 6y = 12$ are the same line b/c the 2nd is exactly the 1st multiplied by 2.

⚠ Slope is Δy over Δx. The reciprocal (Δx/Δy) is almost always one of the wrong answer choices.

✓ Graph given? Pick 2 lattice points (where the line crosses grid corners) to read the slope exactly.

Linear Models in Context

Interpreting m & b

Slope = rate per unit of x: "the cost increases by $12 for each additional ticket." Always include units.
y-intercept = value when x = 0: a starting amount, flat fee or initial height, before any change happens.

Model	Slope means	Intercept means
C = 15h + 40	$15 per hour	$40 fee
W = 500 − 20d	−20 per day	500 at start
T = 0.5m + 68	+0.5° per min	68° at m = 0

Negative slope: the quantity decreases over time, e.g., a tank losing 20 gallons per day: W = 500 − 20d.

Writing the Equation

Translate: "$40 plus $15 per hour" → C = 15h + 40. Rate goes w/ the variable; fixed amount stands alone.
Key phrases: "per" → multiply by the variable; "initial / flat / base" → constant; "at least" → ≥; "total" → =.
Two-part totals: 3a + 5b = 120 means a items at $3 & b items at $5 cost $120 together; a & b are counts.
Rates as fractions: "1 inch per 3 days" → slope 1/3; "3 days per inch" is the reciprocal, a trap choice.

Using the Model

Predict: plug in x to find y. **Reverse**: set y = target & solve for x: 15h + 40 = 145 → h = 7 hours of work.
Change in y: when x increases by k, y changes by m · k (not by m). In C = 15h + 40, 3 more hours → +$45.
Graph of a model: the y-intercept is the starting value; a steeper line means a faster rate of change.
Example: a candle 30 cm tall burns 2 cm per hour → h = 30 − 2t; it burns out when h = 0 at t = 15 hours.

⚠ Interpret the coefficient, not the variable: in C = 15h + 40, "15" is the hourly rate in dollars, not "h."

✓ Check units: if C is dollars & h is hours, then the slope must be dollars per hour & the intercept dollars.

Systems of Linear Equations

Methods

Substitution: solve 1 equation for a variable, then plug that expression into the other equation; best if a coefficient is 1.
Elimination: add or subtract equations (after scaling) so 1 variable cancels; best for standard form.
Example: $2x + 3y = 12$ & $4x - 3y = 6 \rightarrow$ add: $6x = 18 \rightarrow x = 3 \rightarrow 3y = 6 \rightarrow y = 2$. Check (3, 2) in both equations.
Scaling: $3x + 2y = 16$ & $x - y = 2 \rightarrow \times 2$ the 2nd: $2x - 2y = 4 \rightarrow$ add: $5x = 20 \rightarrow x = 4$, y = 2 (check: 12 + 4 = 16).

Number of Solutions

Lines	Slopes & intercepts	Solutions
Intersect	Different slopes	Exactly 1
Parallel	Same m, different b	None
Same line	Same m, same b	Infinitely many

No solution: $ax + 6y = 5$ & $2x + 3y = 9 \rightarrow$ coefficients proportional ($a/2 = 6/3$) → a = 4, constants not.
Infinitely many: one equation is a multiple of the other, constants included ($2x + 4y = 10$ & $x + 2y = 5$).

Shortcuts

Asked for x + y? Add or subtract the equations first: $3x + y = 11$ & $x + 3y = 9 \rightarrow 4x + 4y = 20 \rightarrow x + y = 5$.
Asked for x – y? Subtract the equations: $5x + 2y = 20$ & $4x + 3y = 13 \rightarrow x - y = 7$, no need to solve for each.
Solution point: the solution (x, y) is where the 2 lines cross; it must satisfy BOTH equations at the same time.
Elimination vs. substitution: choose elimination when coefficients line up; substitution when 1 variable is alone.
Fractions in systems: multiply each equation by its own LCD first to clear fractions, then eliminate.

⚠ Solving for x isn't finishing. If the question asks for y, x – y or 2x + y, take the final step before answering.

✓ Desmos: type both equations exactly as given; tap the intersection point to read (x, y) instantly.

Systems in Word Problems

Setting Up

Rule: 1 equation per fact. Usually a COUNT equation & a VALUE equation (money, weight, points).
Tickets: 200 sold, adults $8, students $5, total $1,300 → a + s = 200 (count) & 8a + 5s = 1,300 (money).
Coins: 30 coins of nickels & dimes worth $2.10 → n + d = 30 & 0.05n + 0.10d = 2.10 → d = 12, n = 18.
Solve: s = 200 − a → 8a + 5(200 − a) = 1,300 → 3a = 300 → a = 100 adults & s = 100 students. Check both.

Problem type	Count eq.	Value eq.
Tickets/items	a + b = total	pa + qb = $
Coins	n + d = coins	5n + 10d = ¢
Test questions	x + y = qs	2x + 4y = pts
Mixtures	a + b = amount	ra + sb = total

Reading the Question

Define variables clearly (x = # of adults), & check which one the question actually asks you to find.
Which system? Many SAT questions ask you to CHOOSE the system that models a situation, not solve it.
Match by meaning: the count equation adds items; the value equation multiplies each item by its price.
Percent mixtures: 0.10a + 0.30b = 0.25(a + b) models mixing 10% & 30% solutions to get a 25% mix.

Comparing Plans

Break-even: Plan A = 20 + 5x, Plan B = 50 + 2x → set equal: $3x = 30 \rightarrow x = 10$; equal cost at 10 uses.
Which is cheaper? After the break-even point, the plan w/ the smaller rate (slope) always costs less.

⚠ Don't swap the prices: 8a + 5s, not 5a + 8s. Wrong pairing is a common wrong answer choice.

✓ Check your answer in BOTH original facts (the count & the total) before you move on to the next question.

Linear Inequalities

One Variable

Solve like an equation, but FLIP the sign when you multiply or divide by a negative: $-2x > 8 \rightarrow x < -4$.
Compound: $-3 \leq 2x + 1 < 9 \rightarrow$ apply each step to all 3 parts: $-4 \leq 2x < 8 \rightarrow -2 \leq x < 4$. x can be −2 but not 4.
Integer answers: "greatest integer x" when $x < 4.5 \rightarrow 4$; "least integer x" when $x > -2 \rightarrow -1$ (not −2).
Double inequality from words: "between 40 & 60, inclusive" → $40 \leq x \leq 60$; "exclusive" uses < instead.

Two Variables

Symbol	Line	Shade
y > mx + b	Dashed	Above
y ≥ mx + b	Solid	Above
y < mx + b	Dashed	Below
y ≤ mx + b	Solid	Below

Test a point: plug in (0, 0) or any point; if it makes the inequality true, it's in the solution region.
Find the inequality: read the boundary line's slope & intercept, then check shading & solid/dashed.
Systems: the solution is where the shaded regions OVERLAP; points on a dashed line aren't included.

Word Problems

Key phrases: "at least" → ≥; "at most / no more than" → ≤; "more than" → >; "fewer than" → <; "exceeds" → >.
Example: $500 budget, $40 fee + $25/hour → $40 + 25h \leq 500 \rightarrow h \leq 18.4 \rightarrow$ at most 18 full hours.
Two variables: shirts $15 & hats $10, spend at most $120 → $15s + 10h \leq 120$, w/ $s \geq 0$ & $h \geq 0$ as limits.
Constraints: items can't be negative, so $x \geq 0$ & $y \geq 0$ are often implied; counts must be whole numbers.

⚠ "At least" means ≥, the minimum allowed. Students often reverse it to ≤ under time pressure; reread.

✓ Desmos shades inequalities for you: type them & check which answer point lands in the overlap.

Test Strategy & Desmos

The Math Section

Format: 2 adaptive modules, 22 questions & 35 minutes each; Algebra ≈ 35% (about 13–15 questions).
Adaptive: your Module 1 score sets Module 2's difficulty; strong Module 1 → harder but higher-scoring Module 2.
Calculator: built-in Desmos is allowed on every question; you may also bring an approved calculator.
Other domains: Advanced Math ≈ 35%, Problem-Solving & Data ≈ 15%, Geometry & Trig ≈ 15% of questions.

Student-Produced Responses

Rule	Detail
Length	5 chars (+), 6 (−)
Fractions	OK if they fit: 7/3
Decimals	Fill space or round
Mixed numbers	Not allowed
Multiple answers	Enter just 1

Long decimal: 2/3 → enter 2/3, .6666 or .6667; 0.66 or 0.67 is marked wrong (not precise enough).

Fast Methods

Backsolve: plug answer choices into the equation, starting w/ a middle value, when solving feels slow.
Plug in numbers: for expression answers, pick x = 2, evaluate the original & each choice, & match.
Sliders: for "for what value of k...", type the equation w/ k & move the slider until the condition holds.
Desmos tricks: graph both sides to solve; use sliders for unknown constants; tables for regressions.
Pacing: about 1.5 minutes per question; easy algebra items should take under a minute to bank time.

⚠ No penalty for guessing. Never leave a question blank; flag it, make your best guess & come back later.

✓ Reference sheet covers geometry only. Algebra rules (slope, forms, systems) must be memorized.