ALBERT

Name _____ Date _____

Volume Formula Arguments Worksheet

Geometry · Common Core HSG-GMD.A.1

> Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

Practice

Builds the skill step by step.

1. What does Cavalieri's principle say about two solids with the same height?

   A. If the bases are the same shape, the volumes are equal.

   B. If every cross-section at the same height has the same area, the volumes are equal.

   C. Slanted solids always have less volume.

   D. If they have the same surface area, the volumes are equal.

2. A straight stack of coins has a volume of 30 cm^3. The same coins are pushed into a slanted stack. What is the volume of the slanted stack?

3. A pyramid and a prism have the same base and the same height. What fraction of the prism's volume is the pyramid's volume?

4. A cone and a cylinder have the same base and height. What fraction of the cylinder's volume is the cone's?

5. A cylinder has a base area of 20 square units and a height of 6. What is its volume?

6. A circle of radius 5 is cut into many thin sectors and rearranged into a shape close to a parallelogram. Its base is half the circumference, $k\pi$. What is k?

7. What does rearranging a circle's sectors into a near-parallelogram show?

   A. The circle has no area.

   B. The circumference is $2\pi r$.

   C. The area is $2\pi r$.

   D. The area is half the circumference times the radius, which is πr^2.

8. What is the ratio of any circle's circumference to its diameter?

   A. π

   B. 2π

   C. 2

   D. It depends on the circle.

9. An oblique cylinder and a right cylinder both have radius 3 and height 10. What is the ratio of their volumes?

10. A cube with side 6 can be cut into 3 congruent pyramids that meet at one vertex. What is the volume of each pyramid?

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On Level

Grade-level practice.

1. A pyramid has a base area of 30 square units and a height of 9. What is its volume?

2. A cone has radius 3 and height 8. Its volume is kπ cubic units. What is k?

3. Use Cavalieri's principle to explain why an oblique cylinder has the same volume as a right cylinder with the same radius and height.

4. Regular polygons with more and more sides are inscribed in a circle. What happens to their perimeters?

   A. They get larger than the circumference.

   B. They shrink to zero.

   C. They get closer and closer to the circumference.

   D. They stay the same.

5. A prism and a pyramid each have a base area of 18 and a height of 10. How much greater is the prism's volume?

6. A circle has radius 4. Its area is $k\pi$. What is k?

7. A circle has diameter 10. Its circumference is $k\pi$. What is k?

8. A cone has a volume of 50 cubic units. What is the volume of a cylinder with the same base and height?

9. A regular hexagon inscribed in a circle of radius 1 has a perimeter of 6. Dividing that perimeter by the diameter gives an estimate for π. What is it?

10. A slanted stack of paper has a cross-sectional area of 80 cm^2 at every height and a height of 5 cm. What is its volume?

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Challenge

Multi-step and word problems that stretch it.

1. A square inscribed in a circle of radius 1 has a perimeter of about 5.657. Dividing by the diameter gives an estimate for π. What is it, to the nearest tenth?

2. Give an informal argument for the area of a circle, $A = \pi r^2$, by cutting it into sectors.

3. A cone and a cylinder each have radius 6 and height 10. How much greater is the cylinder's volume? It is $k\pi$ cubic units. What is k?

4. A pyramid has a volume of 120 and a 6-by-5 rectangular base. What is its height?

5. Why does an oblique pyramid have the same volume as a right pyramid with the same base and height?

   A. Because slanting a solid always keeps its surface area.

   B. At each height their cross-sections are congruent, so Cavalieri's principle applies.

   C. They don't have the same volume.

   D. Because both have the same slant height.

6. A cube with side 3 is cut into 3 congruent pyramids. What is the volume of each?

7. A cylinder has radius 2 and height 5. Its volume is $k\pi$ cubic units. What is k?

8. A cylinder's radius is doubled and its height stays the same. By what factor does its volume change?

9. A cone and a cylinder have the same base and height. How many cones of water fill the cylinder?

10. A regular octagon inscribed in a circle of radius 1 has a perimeter of about 6.123. Dividing by the diameter gives an estimate for π. What is it, to the nearest tenth?