ALBERT

Name _____ Date _____

Fundamental Theorem of Algebra Worksheet

Algebra 2 · Common Core HSN-CN.C.9

> (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Practice

Builds the skill step by step.

1. How many complex roots, counting multiplicity, does a polynomial of degree 5 have?

2. How many complex roots, counting multiplicity, does a cubic polynomial have?

3. Which statement is the Fundamental Theorem of Algebra?

   A. Every polynomial has at least one real root.

   B. Every quadratic has two real roots.

   C. Every polynomial of degree $n \geq 1$ has exactly n complex roots, counting multiplicity.

   D. Every polynomial can be factored over the integers.

4. How many complex solutions does $x^2 + 4 = 0$ have?

5. How many solutions does $(x - 2)^2 = 0$ have, counting multiplicity?

6. How many different solutions does $x^2 - 4x + 4 = 0$ have?

7. A polynomial with real coefficients has $3 + i$ as a root. Which must also be a root?

   A. i

   B. $3 - i$

   C. $-3 + i$

   D. $-3 - i$

8. A polynomial of degree 4 with real coefficients has exactly 2 real roots, counting multiplicity. How many non-real roots does it have?

9. How many complex solutions, counting multiplicity, does $x^3 - 8 = 0$ have?

10. Can a cubic polynomial with real coefficients have no real roots?

    A. No: non-real roots come in pairs, so at least one root is real.

    B. Yes, always.

    C. Yes, if its leading coefficient is negative.

    D. Only if it has degree 2.

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On Level

Grade-level practice.

1. The roots of $x^2 + 2x + 2$ are $a \pm bi$ with $b > 0$. What are a and b?

2. A polynomial of degree 6 with real coefficients has the roots $2i$, $1 - i$ and 5, each with multiplicity 1. How many of its roots are still unknown?

3. $x^3 + x = x(x^2 + 1)$. How many of its roots are real?

4. Which number of non-real roots is impossible for a cubic polynomial with real coefficients?

   A. none of these: every number is possible

   B. 0

   C. 2

   D. 1

5. How many roots, counting multiplicity, does $(x - 1)^3(x + 2)$ have?

6. Show that the Fundamental Theorem of Algebra holds for $x^2 - 6x + 10$.

7. A polynomial with real coefficients has roots 2 and 3i. What is the lowest degree it can have?

8. A polynomial with real coefficients has roots 1 + 2i and 4. What is the lowest degree it can have?

9. A degree 5 polynomial with real coefficients has 2 as a double root and i as a root. What must be true of its fifth root?

   A. It is real.

   B. It is i.

   C. It is non-real.

   D. It is 2.

10. How many real roots does $x^4 - 1$ have?

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Challenge

Multi-step and word problems that stretch it.

1. $x^2 + bx + c$ has roots $4 \pm i$. What are b and c?

2. $x^3 + bx^2 + cx + d$ has roots 2, 3i and $-3i$. What are b, c and d?

3. Explain why every quadratic $ax^2 + bx + c$ $(a \neq 0)$ has exactly two complex roots, counting multiplicity.

4. $x^4 + 3x^2 - 4 = (x^2 + 4)(x^2 - 1)$. How many of its roots are non-real?

5. How many real roots, counting multiplicity, can a degree 4 polynomial with real coefficients have?

   A. exactly 2

   B. only 4

   C. 0, 2 or 4

   D. 1 or 3

6. A polynomial with real coefficients has roots $1 + i$ and $2 - i$. What is the lowest degree it can have?

7. $x = 1$ is a root of $x^3 - 3x^2 + 4x - 2$, which factors as $(x - 1)(x^2 - 2x + 2)$. The other roots are $a \pm bi$ with $b > 0$. What are a and b?

8. What is the sum of all the roots of $x^3 - 2x^2 + 9x - 18 = (x - 2)(x^2 + 9)$?

9. Explain why a polynomial with real coefficients can't have exactly one non-real root.

10. What is the product of the roots $2 + 5i$ and $2 - 5i$?